

- 1 I, II, IV en V zijn tweedegraadsvergelijkingen. (de hoogste macht van x is steeds x^2 ; te zien na wegwerken haakjes?)
(III is een eerstegraadsvergelijking en VI is een derdegraadsvergelijking)

* * * ■ **Neem GR - practicum 1 door.** (uitwerkingen aan het eind)

Voorkennis: Ontbinden in factoren blz. 164 (op bladzijde 9 in het boek wordt hiernaar verwezen)

V1a \square $x^2 + 5x = x \cdot (x + 5)$.

V1d \square $5x^2 + 20x = 5x \cdot (x + 4)$.

V1b \square $x^2 + x = x \cdot (x + 1)$.

V1e \square $x^3 - 5x^2 = x^2 \cdot (x - 5)$.

V1c \square $3x^2 - 7x = x \cdot (3x - 7)$.

V1f \square $-3x^2 - 8x = -x \cdot (3x + 8)$.

V2a \square $(x + 3) \cdot (x + 5) = x^2 + 5x + 3x + 15 = x^2 + 8x + 15$. V2b \square $15 = 3 \times 5$ en $8 = 3 + 5$.

V3a \square $x^2 + 5x + 4 = (x + 1) \cdot (x + 4)$.

V3g \square $x^2 - 24x - 52 = (x - 26) \cdot (x + 2)$.

V3b \square $x^2 + 4x - 5 = (x + 5) \cdot (x - 1)$.

V3h \square $x^2 + 1x - 56 = (x + 8) \cdot (x - 7)$.

V3c \square $x^2 - 1x - 30 = (x - 6) \cdot (x + 5)$.

V3i \square $x^2 - 1x - 2 = (x - 2) \cdot (x + 1)$.

V3d \square $x^2 + 7x + 10 = (x + 5) \cdot (x + 2)$.

V3j \square $x^2 - 4x + 3 = (x - 3) \cdot (x - 1)$.

V3e \square $x^2 + 10x + 9 = (x + 9) \cdot (x + 1)$.

V3k \square $x^2 - 4x - 12 = (x - 6) \cdot (x + 2)$.

V3f \square $x^2 + 18x - 19 = (x + 19) \cdot (x - 1)$.

V3l \square $x^2 + 5x - 50 = (x + 10) \cdot (x - 5)$.

V4a \square $6x^2 - 6x = 6x \cdot (x - 1)$.

V4d \square $x^2 + 1x - 56 = (x + 8) \cdot (x - 7)$.

V4b \square $x^2 + 10 - 7x = x^2 - 7x + 10 = (x - 5) \cdot (x - 2)$.

V4e \square $12x^2 + 6x = 6x \cdot (2x + 1)$.

V4c \square $x^2 + x = x \cdot (x + 1)$. (zie V1b)

V4f \square $-3x^2 + x = x \cdot (-3x + 1)$.

V5a \square $x^2 - 12x = x \cdot (x - 12)$.

V5d \square $4x^2 + 8x = 4x \cdot (x + 2)$.

V5b \square $x^2 - 12x + 36 = (x - 6) \cdot (x - 6) = (x - 6)^2$.

V5e \square $x^2 + 8x = x \cdot (x + 8)$.

V5c \square $x^2 - 12x - 28 = (x - 14) \cdot (x + 2)$.

V5f \square $x^2 + 8x - 20 = (x + 10) \cdot (x - 2)$.

2a \square $x^2 + 6 = 5x$

2b \square $x \cdot (x - 1) = 12$

2c \square $2x^2 = 5x$

$x^2 - 5x + 6 = 0$

$x^2 - x = 12$

$2x^2 - 5x = 0$

$(x - 3) \cdot (x - 2) = 0$

$x^2 - x - 12 = 0$

$x \cdot (2x - 5) = 0$

$x - 3 = 0 \vee x - 2 = 0$

$(x - 4) \cdot (x + 3) = 0$

$x = 0 \vee 2x - 5 = 0$

$x = 3 \vee x = 2$.

$x - 4 = 0 \vee x + 3 = 0$

$x = 0 \vee x = 5$

$x = 4 \vee x = -3$.

$x = 0 \vee x = \frac{5}{2} = 2\frac{1}{2}$.

2d \square $x = x^2$

2e \square $x^2 = 11$

2f \square $x^2 + 4 = 1$

$x - x^2 = 0$

$x = \pm\sqrt{11}$

$2x^2 - 5x = 0$

$x \cdot (1 - x) = 0$

$x = \sqrt{11} \vee x = -\sqrt{11}$

$x \cdot (2x - 5) = 0$

$x = 0 \vee 1 - x = 0$

$x = 0 \vee x + 3 = 0$

$x = 0 \vee 2x - 5 = 0$

$x = 0 \vee x = 1$.

$x = 4 \vee x = -3$.

$x = 0 \vee x = \frac{5}{2} = 2\frac{1}{2}$.

$x^2 = -3$ (kan niet)

(een kwadraat kan nooit negatief zijn)
geen oplossingen.

3a \square $3x^2 - 6x = 24$

3b \square $3x^2 - 6x = -3 \cdot (x - 6)$

3c \square $2x^2 - 3x = 2$

$3x^2 - 6x - 24 = 0$

$3x^2 - 6x = -3x + 18$

$2x^2 - 3x - 2 = 0$

$x^2 - 2x - 8 = 0$

$3x^2 - 3x - 18 = 0$

$a = 2, b = -3 \text{ en } c = -2$

$(x - 4) \cdot (x + 2) = 0$

$x^2 - 1x - 6 = 0$

$D = b^2 - 4ac = (-3)^2 - 4 \cdot 2 \cdot -2 = 9 + 16 = 25$

$x - 4 = 0 \vee x + 2 = 0$

$(x - 3) \cdot (x + 2) = 0$

$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{25}}{2 \cdot 2} = \frac{3 \pm 5}{4}$

$x = 4 \vee x = -2$.

$x - 3 = 0 \vee x + 2 = 0$

$x = \frac{3+5}{4} = \frac{8}{4} = 2 \vee x = \frac{3-5}{4} = \frac{-2}{4} = -\frac{1}{2}$.

$x = 3 \vee x = -2$.

$x = 0 \vee x = -\frac{1}{2}$.

3d \square $0,5x^2 - 2x - 6 = 0$

3e \square $x^2 - 3x = 5 \cdot (x - 3)$

3f \square $2x^2 - 5x = 3x$

$x^2 - 4x - 12 = 0$

$x^2 - 3x = 5x - 15$

$2x^2 - 8x = 0$

$(x - 6) \cdot (x + 2) = 0$

$x^2 - 8x + 15 = 0$

$2x \cdot (x - 4) = 0$

$x - 6 = 0 \vee x + 2 = 0$

$(x - 5) \cdot (x - 3) = 0$

$2x = 0 \vee x - 4 = 0$

$x = 6 \vee x = -2$.

$x - 5 = 0 \vee x - 3 = 0$

$x = 0 \vee x = 4$.

$x = 5 \vee x = 3$.

4a \square $6 - x^2 = -2$
 $-x^2 = -8$
 $x^2 = 8$
 $x = \pm\sqrt{8}$
 $x = \sqrt{8} \quad \vee \quad x = -\sqrt{8}.$

4b \square

$2x^2 = 9x + 5$
 $2x^2 - 9x - 5 = 0$
 $a = 2, b = -9$ en $c = -5$
 $D = b^2 - 4ac = (-9)^2 - 4 \cdot 2 \cdot -5 = 81 + 40 = 121$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-9 \pm \sqrt{121}}{2 \cdot 2} = \frac{9 \pm 11}{4} \quad \boxed{11}$
 $x = \frac{9+11}{4} = \frac{20}{4} = 5 \quad \vee \quad x = \frac{9-11}{4} = \frac{-2}{4} = -\frac{1}{2}.$

4c \square

$3 \cdot (x+2)^2 + 5 = 80$
 $3 \cdot (x+2)^2 = 75$
 $(x+2)^2 = 25$
 $(x+2)^2 = \pm\sqrt{25} = \pm 5$
 $x+2 = 5 \quad \vee \quad x+2 = -5$
 $x = 3 \quad \vee \quad x = -7.$

4d \square $\frac{1}{2} \cdot (x-3)^2 - 3 = 5$
 $\frac{1}{2} \cdot (x-3)^2 = 8$
 $(x-3)^2 = 16$
 $(x-3)^2 = \pm\sqrt{16} = \pm 4$
 $x-3 = 4 \quad \vee \quad x-3 = -4$
 $x = 7 \quad \vee \quad x = -1.$

4e \square

$-(2x-1)^2 + 5 = 1$
 $-(2x-1)^2 = -4$
 $(2x-1)^2 = 4$
 $(2x-1)^2 = \pm\sqrt{4} = \pm 2$
 $2x-1 = 2 \quad \vee \quad 2x-1 = -2$
 $2x = 3 \quad \vee \quad 2x = -1$
 $x = 1\frac{1}{2} \quad \vee \quad x = -\frac{1}{2}.$

4f \square $8 - 3 \cdot (4x-5)^2 = 5$

$-3 \cdot (4x-5)^2 = -3$
 $(4x-5)^2 = 1$
 $(4x-5)^2 = \pm\sqrt{1} = \pm 1$
 $4x-5 = 1 \quad \vee \quad 4x-5 = -1$
 $4x = 6 \quad \vee \quad 4x = 4$
 $x = 1\frac{1}{2} \quad \vee \quad x = 1.$

5a \square $x^2 - 5x = 0$
 $x \cdot (x-5) = 0$
 $x = 0 \quad \vee \quad x-5 = 0$
 $x = 0 \quad \vee \quad x = 5.$

5b \square

$x^2 - 5x = 14$
 $x^2 - 5x - 14 = 0$
 $(x-7) \cdot (x+2) = 0$
 $x-7 = 0 \quad \vee \quad x+2 = 0$
 $x = 7 \quad \vee \quad x = -2.$

5c \square $x^2 - 5 = 14$

$x^2 = 19$
 $x = \pm\sqrt{19}$
 $x = \sqrt{19} \quad \vee \quad x = -\sqrt{19}.$

5d \square $x^2 - 5 = 14x$
 $x^2 - 14x - 5 = 0$
 $a = 1, b = -14$ en $c = -5$
 $D = b^2 - 4ac = (-14)^2 - 4 \cdot 1 \cdot -5 = 196 + 20 = 216$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-14 \pm \sqrt{216}}{2 \cdot 1} = \frac{14 \pm \sqrt{216}}{2} \quad \boxed{\begin{matrix} (-14)^2 - 4 \cdot 1 \cdot -5 \\ \sqrt{216} \\ 216 \\ 14 \cdot 69693846 \end{matrix}}$
 $x = \frac{14 + \sqrt{216}}{2} \quad \vee \quad x = \frac{14 - \sqrt{216}}{2}.$

5e \square $(2x-1) \cdot (3x+6) = 0$
 $2x-1 = 0 \quad \vee \quad 3x+6 = 0$
 $2x = 1 \quad \vee \quad 3x = -6$
 $x = \frac{1}{2} \quad \vee \quad x = -2.$

5f \square $(2x-1) \cdot (3x+6) = 9x$
 $6x^2 + 12x - 3x - 6 = 9x$
 $6x^2 + 9x - 6 = 9x$
 $6x^2 - 6 = 0$
 $6x^2 = 6$
 $x^2 = 1$
 $x = \pm\sqrt{1} = \pm 1$
 $x = 1 \quad \vee \quad x = -1.$

5g \square $(2x-1) \cdot 3x = 6$

$6x^2 - 3x - 6 = 0$
 $2x^2 - 1x - 2 = 0$
 $a = 2, b = -1$ en $c = -2$
 $D = (-1)^2 - 4 \cdot 2 \cdot -2 = 1 + 16 = 17$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{17}}{2 \cdot 2} = \frac{1 \pm \sqrt{17}}{4}$
 $x = \frac{1 + \sqrt{17}}{4} \quad \vee \quad x = \frac{1 - \sqrt{17}}{4}.$

5h \square $(2x-1) \cdot 3x = 6 - 9x$

$6x^2 - 3x - 6 + 9x = 0$
 $6x^2 + 6x - 6 = 0$
 $x^2 + 1x - 1 = 0$
 $a = 1, b = 1$ en $c = -1$
 $D = 1^2 - 4 \cdot 1 \cdot -1 = 1 + 4 = 5$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{5}}{2 \cdot 1} = \frac{-1 \pm \sqrt{5}}{2}$
 $x = \frac{-1 + \sqrt{5}}{2} \quad \vee \quad x = \frac{-1 - \sqrt{5}}{2}.$

6a $(x+3)^2 = 16x$
 $(x+3) \cdot (x+3) = 16x$
 $x^2 + 3x + 3x + 9 = 16x$
 $x^2 - 10x + 9 = 0$
 $(x-9) \cdot (x-1) = 0$
 $x = 9 \quad \vee \quad x = 1.$

6b

$(2x+3)^2 = -16$ kan niet
(een kwadraat kan niet negatief zijn)
geen oplossingen.

6c

$2(x+3)^2 = -4x$
 $(x+3) \cdot (x+3) = -2x$
 $x^2 + 3x + 3x + 9 = -2x$
 $x^2 + 8x + 9 = 0$
 $a = 1, b = 8$ en $c = 9$
 $D = 8^2 - 4 \cdot 1 \cdot 9 = 64 - 36 = 28$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-8 \pm \sqrt{28}}{2 \cdot 1} = \frac{-8 \pm \sqrt{28}}{2}$
 $x = \frac{-8 + \sqrt{28}}{2} \quad \vee \quad x = \frac{-8 - \sqrt{28}}{2}.$

6d $(2x+3) \cdot (4-x) = 9$
 $8x - 2x^2 + 12 - 3x = 9$
 $-2x^2 + 5x + 3 = 0$
 $a = -2, b = 5$ en $c = 3$
 $D = 5^2 - 4 \cdot -2 \cdot 3 = 25 + 24 = 49$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{49}}{2 \cdot -2} = \frac{-5 \pm 7}{4} = \frac{5 \mp 7}{4}$
 $x = \frac{5+7}{4} = \frac{12}{4} = 3 \quad \vee \quad x = \frac{5-7}{4} = \frac{-2}{4} = -\frac{1}{2}.$

6e

$(-4x+3)^2 = 36$
 $-4x+3 = \pm\sqrt{36} = \pm 6$
 $-4x+3 = 6 \quad \vee \quad -4x+3 = -6$
 $-4x = 3 \quad \vee \quad -4x = -9$
 $x = \frac{3}{-4} = -\frac{3}{4} \quad \vee \quad x = \frac{-9}{-4} = 2\frac{1}{4}.$

6f $-4(x+3)^2 = 4x$
 $(x+3)^2 = -x$
 $x^2 + 6x + 9 = -x$
 $x^2 + 7x + 9 = 0$
 $a = 1, b = 7 \text{ en } c = 9$
 $D = 7^2 - 4 \cdot 1 \cdot 9 = 49 - 36 = 13$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-7 \pm \sqrt{13}}{2 \cdot 1} = \frac{-7 \pm \sqrt{13}}{2}$
 $x = \frac{-7 + \sqrt{13}}{2} \quad \vee \quad x = \frac{-7 - \sqrt{13}}{2}.$

6g $x^2 - (x+1)^2 = (x+3)^2$
 $x^2 - (x^2 + 2x + 1) = x^2 + 6x + 9$
 $-2x - 1 = x^2 + 6x + 9$
 $-x^2 - 8x - 10 = 0$
 $x^2 + 8x + 10 = 0$
 $a = 1, b = 8 \text{ en } c = 10$
 $D = 8^2 - 4 \cdot 1 \cdot 10 = 64 - 40 = 24$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-8 \pm \sqrt{24}}{2 \cdot 1} = \frac{-8 \pm \sqrt{24}}{2}$
 $x = \frac{-8 + \sqrt{24}}{2} \quad \vee \quad x = \frac{-8 - \sqrt{24}}{2}.$

6h $(x+3)^2 + (x+2)^2 = 25$
 $x^2 + 6x + 9 + x^2 + 4x + 4 = 25$
 $2x^2 + 10x - 12 = 0$
 $x^2 + 5x - 6 = 0$
 $(x+6) \cdot (x-1) = 0$
 $x = -6 \quad \vee \quad x = 1.$

7a $x^2 - 1x - 6 = 0$
 $(x-3) \cdot (x+2) = 0$
 $x = 3 \quad \vee \quad x = -2.$

7b $x^2 + 2x - 6 = 0$
 $a = 1, b = 2 \text{ en } c = -6$
 $D = 2^2 - 4 \cdot 1 \cdot -6 = 4 + 24 > 0$
dus 2 oplossingen.

7c $x^2 + px - 6 = 0$
 $a = 1, b = p \text{ en } c = -6$
 $D = p^2 - 4 \cdot 1 \cdot -6 = p^2 + 24 \geq 24 > 0$
dus 2 oplossingen.

8a $a = 1, b = -7 \text{ en } c = p$
 $D = (-7)^2 - 4 \cdot 1 \cdot p = 49 - 4p$
2 oplossingen $\Rightarrow D = 49 - 4p > 0$
 $-4p > -49$
 $p < \frac{-49}{-4} = 12\frac{1}{4}.$

8b $a = 2, b = -5 \text{ en } c = -p$
 $D = (-5)^2 - 4 \cdot 2 \cdot -p = 25 + 8p$
2 oplossingen $\Rightarrow D = 25 + 8p > 0$
 $8p > -25$
 $p > \frac{-25}{8} = -3\frac{1}{8}.$

8c $a = -3, b = 4 \text{ en } c = -p$
 $D = 4^2 - 4 \cdot -3 \cdot -p = 16 - 12p$
twee oplossingen $\Rightarrow D = 16 - 12p > 0$
 $-12p > -16$
 $p < \frac{-16}{-12} = 1\frac{1}{3}.$

8d $a = \frac{1}{4}, b = -3 \text{ en } c = p$
 $D = (-3)^2 - 4 \cdot \frac{1}{4} \cdot p = 9 - p$
twee oplossingen $\Rightarrow D = 9 - p > 0$
 $-p > -9$
 $p < \frac{-9}{-1} = 9.$

9a $a = 1, b = p \text{ en } c = 25$
 $D = p^2 - 4 \cdot 1 \cdot 25 = p^2 - 100$
twee opl. $\Rightarrow D = p^2 - 100 > 0 \quad \odot$
 $p^2 > 100$
 $p < -10 \quad \vee \quad p > 10.$

9b $a = 1, b = p \text{ en } c = 4$
 $D = p^2 - 4 \cdot 1 \cdot 4 = p^2 - 16$
geen opl. $\Rightarrow D = p^2 - 16 < 0 \quad \odot$
 $p^2 < 16$
 $-4 < p < 4.$

9c $a = -2, b = p \text{ en } c = 3$
 $D = p^2 - 4 \cdot -2 \cdot 3 = p^2 + 24 > 0.$
(dus voor elke p twee oplossingen)

10a $1^2 + 2 \cdot 1 + p = 0 \Rightarrow 3 + p = 0 \Rightarrow p = -3.$
De vergelijking is: $x^2 + 2x - 3 = 0$
 $(x+3) \cdot (x-1) = 0$
 $x = -3 \quad \vee \quad x = 1$ (was bekend).

10b $p^2 - 11 \cdot 2 + 10 = 0 \Rightarrow 4p - 12 = 0 \Rightarrow 4p = 12 \Rightarrow p = 3.$
Dus $3x^2 - 11x + 10 = 0$ ($a = 3, b = -11 \text{ en } c = 10$)
 $D = (-11)^2 - 4 \cdot 3 \cdot 10 = 121 - 120 = 1$ $\frac{(-11)^2 - 4 \cdot 3 \cdot 10}{1}$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{11 \pm \sqrt{1}}{2 \cdot 3} = \frac{11 \pm 1}{6}$
 $x = \frac{11+1}{6} = \frac{12}{6} = 2$ (bekend) $\quad \vee \quad x = \frac{11-1}{6} = \frac{10}{6} = \frac{5}{3} = 1\frac{2}{3}.$

11a De vergelijking is: $0 + 3x + 1 = 0$; deze heeft één oplossing (het is een eerstegraadsvergelijking).

11b $px^2 + 3x + 1 = 0$ ($a = p \neq 0, b = 3 \text{ en } c = 1$) $\Rightarrow D = 3^2 - 4 \cdot p \cdot 1 = 9 - 4p;$
twee oplossingen $\Rightarrow D = 9 - 4p > 0 \Rightarrow -4p > -9 \Rightarrow p < \frac{9}{4}$. Dus $p < \frac{9}{4}$ én $p \neq 0 \Rightarrow p < 0 \quad \vee \quad 0 < p < 2\frac{1}{4}.$

12a $px^2 + 5x + 2 = 0$ ($a = p \neq 0, b = 5 \text{ en } c = 2$) $\Rightarrow D = 5^2 - 4 \cdot p \cdot 2 = 25 - 8p;$
twee oplossingen $\Rightarrow D = 25 - 8p > 0 \Rightarrow -8p > -25 \Rightarrow p < \frac{25}{8}$. Dus $p < \frac{25}{8}$ én $p \neq 0 \Rightarrow p < 0 \quad \vee \quad 0 < p < 3\frac{1}{8}.$

12b $px^2 - 3x - 4 = 0$ ($a = p \neq 0, b = -3 \text{ en } c = -4$) $\Rightarrow D = (-3)^2 - 4 \cdot p \cdot -4 = 9 + 16p;$
twee oplossingen $\Rightarrow D = 9 + 16p > 0 \Rightarrow 16p > -9 \Rightarrow p > \frac{-9}{16}$. Dus $p > -\frac{9}{16}$ én $p \neq 0 \Rightarrow -\frac{9}{16} < p < 0 \quad \vee \quad p > 0.$

13a $2x^2 + x + p = 0$ ($a = 2, b = 1 \text{ en } c = p$); geen oplossing $\Rightarrow D = 1^2 - 4 \cdot 2 \cdot p = 1 - 8p < 0 \Rightarrow -8p < -1 \Rightarrow p > \frac{-1}{8} \Rightarrow p > \frac{1}{8}.$

13b $px^2 + x + p = 0$ ($a = p \neq 0$, $b = 1$ en $c = p$) ($p = 0$ geeft 1 oplossing, namelijk $x = 0$);
twee oplossingen $\Rightarrow D = 1^2 - 4 \cdot p \cdot p = 1 - 4p^2 > 0 \Rightarrow -4p^2 > -1 \Rightarrow 4p^2 < 1 \Rightarrow p^2 < \frac{1}{4} \Leftrightarrow -\frac{1}{2} < p < \frac{1}{2}$ én $p \neq 0$.

13c $2x^2 + px + 1 = 0$ ($a = 2$, $b = p$ en $c = 1$);
twee oplossingen $\Rightarrow D = p^2 - 4 \cdot 2 \cdot 1 = p^2 - 8 > 0 \Rightarrow p^2 > 8 \Rightarrow$ (grafiek is dalparabool) $\Rightarrow p < -\sqrt{8} \vee p > \sqrt{8}$.

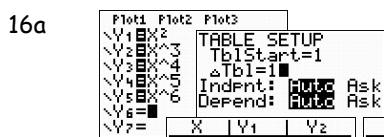
14a $p = 0 \Rightarrow 6x + 9 = 0 \Rightarrow 6x = -9 \Rightarrow x = -\frac{9}{6} = -\frac{3}{2} = -1\frac{1}{2}$.
 $p \neq 0 \Rightarrow px^2 + 6x + 9 = 0$ ($a = p \neq 0$, $b = 6$ en $c = 9$);

één oplossing $\Rightarrow D = 6^2 - 4 \cdot p \cdot 9 = 36 - 36p = 0 \Rightarrow -36p = -36 \Rightarrow p = 1 \Rightarrow x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-6 \pm \sqrt{0}}{2 \cdot 1} = \frac{-6}{2} = -3$.

14b $x^2 + px + 1 = 0$ ($a = 1$, $b = p$ en $c = 1$);
één oplossing $\Rightarrow D = p^2 - 4 \cdot 1 \cdot 1 = p^2 - 4 = 0 \Rightarrow p^2 = 4 \Rightarrow p = \pm 2$.
 $p = b = 2 \Rightarrow x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-2 \pm \sqrt{0}}{2 \cdot 1} = \frac{-2}{2} = -1$ en $p = b = -2 \Rightarrow x = \frac{-b \pm \sqrt{D}}{2a} = \frac{2 \pm \sqrt{0}}{2 \cdot 1} = \frac{2}{2} = 1$.

15a $x^3 = 10$ heeft één oplossing, omdat de grafiek van f en de horizontale lijn $y = 10$ één snijpunt hebben. (zie fig. 1.1a)
 $x^3 = -10$ heeft ook één oplossing, omdat de grafiek van f en de horizontale lijn $y = -10$ één snijpunt hebben.

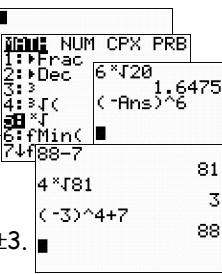
15b $x^4 = 10$ heeft twee oplossingen, omdat de grafiek van f en de lijn $y = 10$ twee snijpunten hebben. (zie figuur 1.1b)
 $x^4 = -10$ heeft geen oplossingen, omdat de grafiek van f en de lijn $y = -10$ geen snijpunten hebben.



16b * $\begin{array}{|c|c|c|} \hline X & Y_1 & Y_2 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \begin{array}{|c|c|c|} \hline X & Y_3 & Y_4 \\ \hline 1 & 1 & 1 \\ \hline \end{array} \quad \begin{array}{|c|c|c|} \hline X & Y_5 \\ \hline 1 & 1 \\ \hline \end{array}$

x	x^2	x^3	x^4	x^5	x^6
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	
5	25	125	625		
6	36	216			
7	49	343			
8	64				
9	81				

17a $x^6 = 20$
 $x = \pm \sqrt[6]{20}$.



17b $5x^3 = 135$

$x^3 = 27$
 $x = \sqrt[3]{27} = 3$

17c $0,5x^5 = 20$
 $x^5 = 40$
 $x = \sqrt[5]{40}$

17d $x^4 + 7 = 88$
 $x^4 = 81$
 $x = \pm \sqrt[4]{81} = \pm 3$

17e $1 - 3x^5 = 97$
 $-3x^5 = 96$
 $x^5 = -32$
 $x = \sqrt[5]{-32} = -2$

17f $\frac{1}{4}x^8 + 3 = 10$
 $\frac{1}{4}x^8 = 7$
 $x^8 = 28$
 $x = \pm \sqrt[8]{28}$

18a $5x^4 - 1 = 4$
 $5x^4 = 5$
 $x^4 = 1$
 $x = \pm \sqrt[4]{1} = \pm 1$

18b $5x^4 = -4$
 $x^4 = -\frac{4}{5} = -\frac{4}{5}$
geen oplossing.

18c $5x^3 - 1 = 9$
 $5x^3 = 10$
 $x^3 = 2$
 $x = \sqrt[3]{2}$

18d $8x^3 + 2 = 1$
 $8x^3 = -1$
 $x^3 = -\frac{1}{8} = -\frac{1}{8}$
 $x = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$.

18e $5x^6 + 7 = 97$
 $5x^6 = 90$
 $x^6 = 18$
 $x = \pm \sqrt[6]{18}$.

18f $0,1x^7 - 1 = 999$
 $0,1x^7 = 1000$
 $x^7 = 10000$
 $x = \sqrt[7]{10000}$

19a $3(x-2)^4 + 7 = 37$
 $3(x-2)^4 = 30$
 $(x-2)^4 = 10$
 $x-2 = \pm \sqrt[4]{10}$
 $x = 2 \pm \sqrt[4]{10}$
 $x = 2 + \sqrt[4]{10} \vee x = 2 - \sqrt[4]{10}$.

19b $6 - (2x-1)^3 = 1$
 $-(2x-1)^3 = -5$
 $(2x-1)^3 = 5$
 $2x-1 = \sqrt[3]{5}$
 $2x = 1 + \sqrt[3]{5}$
 $x = \frac{1}{2} + \frac{1}{2} \cdot \sqrt[3]{5}$.

17a $x = \pm \sqrt[6]{20}$

17b $x^3 = 27$

17c $x^5 = 40$

17d $x = \pm \sqrt[4]{81} = \pm 3$

17e $x^5 = -32$

17f $x = \pm \sqrt[5]{-32} = \pm 2$

18a $x = \pm 1$

18b $x = \pm \sqrt[4]{-\frac{4}{5}} = \text{no solution}$

18c $x = \pm \sqrt[3]{2}$

18d $x = \pm \sqrt[3]{-\frac{1}{8}} = \pm \frac{1}{2}$

18e $x = \pm \sqrt[6]{18}$

18f $x = \pm \sqrt[7]{10000}$

19a $x = \pm \sqrt[4]{10}$

19b $x = \pm \sqrt[3]{5}$

19c $x = \pm \sqrt[3]{5}$

19d $x = \pm \sqrt[3]{5}$

19e $x = \pm \sqrt[3]{5}$

19f $x = \pm \sqrt[3]{5}$

19c $\frac{1}{2}(3x-1)^4 = 8$

$$(3x-1)^4 = 16$$

$$3x-1 = \pm\sqrt[4]{16} = \pm 2$$

$$3x = 1 \pm 2$$

$$x = \frac{1 \pm 2}{3}$$

$$x = \frac{1+2}{3} = 1 \quad \vee \quad x = \frac{1-2}{3} = -\frac{1}{3}.$$

$8/(1/2)$	16
$4\sqrt{16}$	2
$(1-2)/3$	-1/3

19d $100 - \frac{1}{3}(4x-3)^5 = 19$

$$-\frac{1}{3}(4x-3)^5 = -81$$

$$(4x-3)^5 = 243$$

$$4x-3 = \sqrt[5]{243} = 3$$

$$4x = 6$$

$$x = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}.$$

19-100	-81
$Ans/(-1/3)$	243
$5\sqrt{243}$	3
$3+3$	6
$6/4$	1.5
$Ans\rightarrow\text{Frac}$	3/2
■	■

20a $5x^4 - 3 = 17$

$$5x^4 = 20$$

$$x^4 = 4$$

$$x = \pm\sqrt[4]{4}.$$

$17+3$	20
$Ans/5$	4
$4\sqrt{4}$	1.414213562
■	■

20b $4x^3 - 5 = 1367$

$$4x^3 = 1372$$

$$x^3 = 343$$

$$x = \sqrt[3]{343} = 7.$$

$1367+5$	1372
$1372/4$	343
$3\sqrt{343}$	7
■	■

20c $3(4x-5)^3 = 15$

$$(4x-5)^3 = 5$$

$$4x-5 = \sqrt[3]{5}$$

$$4x = 5 + \sqrt[3]{5}$$

$$x = \frac{5}{4} + \frac{1}{4} \cdot \sqrt[3]{5} = 1\frac{1}{4} + \frac{1}{4} \cdot \sqrt[3]{5}.$$

$15/3$	5
$3\sqrt{5}$	1.709975947
■	■

20d $17 - 2(1-3x)^4 = 5$

$$-2(1-3x)^4 = -12$$

$$(1-3x)^4 = 6$$

$$1-3x = \pm\sqrt[4]{6}$$

$$-3x = -1 \pm \sqrt[4]{6}$$

$$x = \frac{1}{3} \mp \frac{1}{3} \cdot \sqrt[4]{6} = \frac{1}{3} \pm \frac{1}{3} \cdot \sqrt[4]{6}.$$

$5-17$	-12
$-12/-2$	6
$4\sqrt{6}$	1.56508458
■	■

21ab $x^3 - x^2 - 2x = x \cdot (x^2 - x - 2) = x \cdot (x-2) \cdot (x+1) = 0 \Rightarrow x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -1.$

■

22a $x^3 - 5x^2 + 6x = 0$

$$x \cdot (x^2 - 5x + 6) = 0$$

$$x \cdot (x-3) \cdot (x-2) = 0$$

$$x = 0 \quad \vee \quad x = 3 \quad \vee \quad x = 2.$$

22b $x^3 - 5x^2 = 6x$

$$x^3 - 5x^2 - 6x = 0$$

$$x \cdot (x^2 - 5x - 6) = 0$$

$$x \cdot (x-6) \cdot (x+1) = 0$$

$$x = 0 \quad \vee \quad x = 6 \quad \vee \quad x = -1.$$

22c $x^3 = 4x^2 + 12x$

$$x^3 - 4x^2 - 12x = 0$$

$$x \cdot (x^2 - 4x - 12) = 0$$

$$x \cdot (x-6) \cdot (x+2) = 0$$

$$x = 0 \quad \vee \quad x = 6 \quad \vee \quad x = -2.$$

22d $x^4 - 13x^2 + 36 = 0$ (noem x^2 tijdelijk t)

$$t^2 - 13t + 36 = 0$$

$$(t-4) \cdot (t-9) = 0$$

$$t = x^2 = 4 \quad \vee \quad t = x^2 = 9$$

$$x = \pm 2 \quad \vee \quad x = \pm 3.$$

23a $x^4 - 10x^2 + 9 = 0$ (noem x^2 tijdelijk t)

$$t^2 - 10t + 9 = 0$$

$$(t-9) \cdot (t-1) = 0$$

$$t = x^2 = 9 \quad \vee \quad t = x^2 = 1$$

$$x = \pm 3 \quad \vee \quad x = \pm 1.$$

23b $x^4 - 8x^2 - 9 = 0$ (noem x^2 tijdelijk t)

$$t^2 - 8t - 9 = 0$$

$$(t-9) \cdot (t+1) = 0$$

$$t = x^2 = 9 \quad \vee \quad t = x^2 = -1$$
 (kan niet)
$$x = \pm 3.$$

23c $x^4 + 16 = 10x^2$

$$x^4 - 10x^2 + 16 = 0$$
 (noem x^2 tijdelijk t)
$$t^2 - 10t + 16 = 0$$

$$(t-8) \cdot (t-2) = 0$$

$$t = x^2 = 8 \quad \vee \quad t = x^2 = 2$$

$$x = \pm\sqrt{8} \quad \vee \quad x = \pm\sqrt{2}.$$

23d $x^3 + 25x = 10x^2$

$$x^3 - 10x^2 - 25x = 0$$

$$x \cdot (x^2 - 10x - 25) = 0$$

$$x \cdot (x-5) \cdot (x-5) = 0$$

$$x = 0 \quad \vee \quad x = 5$$
 (dubbel).

24ab $2x^4 - 11x^2 + 12 = 0$ (noem x^2 tijdelijk p)

$$2p^2 - 11p + 12 = 0$$
 ($a = 2$, $b = -11$ en $c = 12$)
$$D = (-11)^2 - 4 \cdot 2 \cdot 12 = 121 - 96 = 25$$

$$p = \frac{-b \pm \sqrt{D}}{2a} = \frac{11 \pm \sqrt{25}}{2 \cdot 2} = \frac{11 \pm 5}{4}$$

$$p = x^2 = \frac{11+5}{4} = \frac{16}{4} = 4 \quad \vee \quad p = x^2 = \frac{11-5}{4} = \frac{6}{4} = \frac{3}{2} = 1\frac{1}{2}$$

$$x = \pm 2 \quad \vee \quad x = \pm\sqrt{1\frac{1}{2}}.$$

$(-11)^2-4*2*12$	25
$\sqrt{1.5}\rightarrow\text{Frac}$	1.224744871
■	■

■

25a ■ $6x^4 + 2 = 7x^2$

$$6x^4 - 7x^2 + 2 = 0 \text{ (noem } x^2 \text{ tijdelijk } t)$$

$$6t^2 - 7t + 2 = 0 \text{ (a} = 6, b = -7 \text{ en } c = 2\text{)}$$

$$D = (-7)^2 - 4 \cdot 6 \cdot 2 = 49 - 48 = 1 \quad \boxed{(-7)^2 - 4 \cdot 6 \cdot 2}$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{7 \pm \sqrt{1}}{2 \cdot 6} = \frac{7 \pm 1}{12}$$

$$t = x^2 = \frac{7+1}{12} = \frac{8}{12} = \frac{2}{3} \quad \vee \quad t = x^2 = \frac{7-1}{12} = \frac{6}{12} = \frac{1}{2}$$

$$x = \pm \sqrt{\frac{2}{3}} \quad \vee \quad x = \pm \sqrt{\frac{1}{2}}.$$

25c ■ $4x^4 + 7x^2 = 2$

$$4x^4 + 7x^2 - 2 = 0 \text{ (noem } x^2 \text{ tijdelijk } t)$$

$$4t^2 + 7t - 2 = 0 \text{ (a} = 4, b = 7 \text{ en } c = -2\text{)}$$

$$D = 7^2 - 4 \cdot 4 \cdot -2 = 49 + 32 = 81$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{-7 \pm \sqrt{81}}{2 \cdot 4} = \frac{-7 \pm 9}{8}$$

$$t = x^2 = \frac{-7+9}{8} = \frac{2}{8} = \frac{1}{4} \quad \vee \quad t = x^2 = \frac{-7-9}{8} = \frac{-16}{8} = -2 \text{ (k.n.)}$$

$$x = \pm \sqrt{\frac{1}{4}} = \pm \frac{1}{2}.$$

26a $4x^4 + 153 = 53x^2$

$$4x^4 - 53x^2 + 153 = 0 \text{ (noem } x^2 \text{ tijdelijk } t)$$

$$4t^2 - 53t + 153 = 0 \text{ (a} = 4, b = -53 \text{ en } c = 153\text{)}$$

$$D = (-53)^2 - 4 \cdot 4 \cdot 153 = 361 \quad \boxed{(-53)^2 - 4 \cdot 4 \cdot 153}$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{53 \pm \sqrt{361}}{2 \cdot 4} = \frac{53 \pm 19}{8} \quad \boxed{53 \pm 19}$$

$$x^2 = \frac{53+19}{8} = \frac{72}{8} = 9 \quad \vee \quad x^2 = \frac{53-19}{8} = \frac{34}{8} = \frac{17}{4} = 4 \frac{1}{4}$$

$$x = \pm 3 \quad \vee \quad x = \pm \sqrt{4 \frac{1}{4}}.$$

26c $4x^6 + 35 = 24x^3$

$$4x^6 - 24x^3 + 35 = 0 \text{ (noem } x^3 \text{ tijdelijk } t)$$

$$4t^2 - 24t + 35 = 0 \text{ (a} = 4, b = -24 \text{ en } c = 35\text{)}$$

$$D = (-24)^2 - 4 \cdot 4 \cdot 35 = 16 \quad \boxed{(-24)^2 - 4 \cdot 4 \cdot 35}$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{24 \pm \sqrt{16}}{2 \cdot 4} = \frac{24 \pm 4}{8} \quad \boxed{24 \pm 4}$$

$$x^3 = \frac{24+4}{8} = \frac{28}{8} = \frac{7}{2} \quad \vee \quad x^3 = \frac{24-4}{8} = \frac{20}{8} = \frac{5}{2}$$

$$x = \sqrt[3]{\frac{7}{2}} \quad \vee \quad x = \sqrt[3]{\frac{5}{2}}.$$

27a De getallen 7 en -7.

25b ■ $2x^4 = x^2 + 3$

$$2x^4 - x^2 - 3 = 0 \text{ (noem } x^2 \text{ tijdelijk } t)$$

$$2t^2 - t - 3 = 0 \text{ (a} = 2, b = -1 \text{ en } c = -3\text{)}$$

$$D = (-1)^2 - 4 \cdot 2 \cdot -3 = 1 + 24 = 25$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{1 \pm \sqrt{25}}{2 \cdot 2} = \frac{1 \pm 5}{4}$$

$$t = x^2 = \frac{1+5}{4} = \frac{6}{4} = 1 \frac{1}{2} \quad \vee \quad t = x^2 = \frac{1-5}{4} = -1 \text{ (k.n.)}$$

$$x = \pm \sqrt{1 \frac{1}{2}}.$$

25d ■ $16x^4 + 225 = 136x^2$

$$16x^4 - 136x^2 + 225 = 0 \text{ (noem } x^2 \text{ tijdelijk } t)$$

$$16t^2 - 136t + 225 = 0 \text{ (a} = 16, b = -136 \text{ en } c = 225\text{)}$$

$$D = (-136)^2 - 4 \cdot 16 \cdot 225 = 4096 \quad \boxed{((-136)^2 - 4 \cdot 16 \cdot 225)}$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{136 \pm \sqrt{4096}}{2 \cdot 16} = \frac{136 \pm 64}{32} \quad \boxed{136 \pm 64}$$

$$t = x^2 = \frac{136+64}{32} = \frac{200}{32} = \frac{25}{4} \quad \vee \quad t = x^2 = \frac{136-64}{32} = \frac{72}{32} = \frac{9}{4}$$

$$x = \pm \sqrt{\frac{25}{4}} = \pm \frac{5}{2} = \pm 2 \frac{1}{2} \quad \vee \quad x = \pm \sqrt{\frac{9}{4}} = \pm \frac{3}{2} = \pm 1 \frac{1}{2}.$$

26b $4x^4 + 21x^2 = 148$

$$4x^4 + 21x^2 - 148 = 0 \text{ (noem } x^2 \text{ tijdelijk } t)$$

$$4t^2 + 21t - 148 = 0 \text{ (a} = 4, b = 21 \text{ en } c = -148\text{)}$$

$$D = 21^2 - 4 \cdot 4 \cdot -148 = 2809$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{-21 \pm \sqrt{2809}}{2 \cdot 4} = \frac{-21 \pm 53}{8} \quad \boxed{-21 \pm 53}$$

$$x^2 = \frac{-21+53}{8} = \frac{32}{8} = 4 \quad \vee \quad x^2 = \frac{-21-53}{8} = -... \text{ (k.n.)}$$

$$x = \pm 2.$$

$$\begin{array}{r} 21^2 - 4 \cdot 4 \cdot -148 \\ \hline 2809 \\ \boxed{2809} \end{array}$$

$$\begin{array}{r} 4 \\ (-21+53)/8 \\ \hline 53 \\ \boxed{53} \end{array}$$

$$\begin{array}{r} 21^2 - 4 \cdot 4 \cdot -148 \\ \hline 2809 \\ \boxed{2809} \end{array}$$

26d $64x^6 + 27 = 224x^3$

$$64x^6 - 224x^3 + 27 = 0 \text{ (noem } x^3 \text{ tijdelijk } t)$$

$$64t^2 - 224t + 35 = 0 \text{ (a} = 64, b = -224 \text{ en } c = 27\text{)}$$

$$D = (-224)^2 - 4 \cdot 64 \cdot 27 = 43264 \quad \boxed{(-224)^2 - 4 \cdot 64 \cdot 27}$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{224 \pm \sqrt{43264}}{2 \cdot 64} = \frac{224 \pm 208}{128} \quad \boxed{224 \pm 208}$$

$$x^3 = \frac{224+208}{128} = \frac{432}{128} = \frac{27}{8} \quad \vee \quad x^3 = \frac{224-208}{128} = \frac{16}{128} = \frac{1}{8}$$

$$x = \sqrt[3]{\frac{27}{8}} = \frac{3}{2} \quad \vee \quad x = \sqrt[3]{\frac{1}{8}} = \frac{1}{2}.$$

$$\begin{array}{r} 224+208/128 \rightarrow Fr \\ ac \\ \hline 27/8 \end{array}$$

$$\begin{array}{r} (224-208)/128 \rightarrow Fr \\ ac \\ \hline 1/8 \end{array}$$

27b $2x - 1 = 7 \quad \vee \quad 2x - 1 = -7$

$$2x = 8 \quad \vee \quad 2x = -6$$

$$x = 4 \quad \vee \quad x = -3.$$

■

28a ■ $|2x - 1| = 8$

$$2x - 1 = 8 \quad \vee \quad 2x - 1 = -8$$

$$2x = 9 \quad \vee \quad 2x = -7$$

$$x = \frac{9}{2} = 4 \frac{1}{2} \quad \vee \quad x = \frac{-7}{2} = -3 \frac{1}{2}.$$

28b ■ $|x^2 - 3| = 1$

$$x^2 - 3 = 1 \quad \vee \quad x^2 - 3 = -1$$

$$x^2 = 4 \quad \vee \quad x^2 = 2$$

$$x = \pm 2 \quad \vee \quad x = \pm \sqrt{2}.$$

28c ■ $|2x^2 - 5| = 11$

$$2x^2 - 5 = 11 \quad \vee \quad 2x^2 - 5 = -11$$

$$2x^2 = 16 \quad \vee \quad 2x^2 = -6$$

$$x^2 = 8 \quad \vee \quad x^2 = -3 \text{ (k.n.)}$$

$$x = \pm \sqrt{8}.$$

28d ■ $|5 - x^2| = 11$

$$5 - x^2 = 11 \quad \vee \quad 5 - x^2 = -11$$

$$-x^2 = 6 \quad \vee \quad -x^2 = -16$$

$$x^2 = -6 \text{ (k.n.)} \quad \vee \quad x^2 = 16$$

$$x = \pm 4.$$

29a $|2x^4 - 5| = 15$
 $2x^4 - 5 = 15 \vee 2x^4 - 5 = -15$
 $2x^4 = 20 \vee 2x^4 = -10$
 $x^4 = 10 \vee x^4 = -5$ (k.n.)
 $x = \pm\sqrt[4]{10}.$

29c $|x^4 - 5x^2| = 6$
 $x^4 - 5x^2 = 6 \vee x^4 - 5x^2 = -6$
 $x^4 - 5x^2 - 6 = 0 \vee x^4 - 5x^2 + 6 = 0$ (noem x^2 tijdelijk t)
 $t^2 - 5t - 6 = 0 \vee t^2 - 5t + 6 = 0$
 $(t-6) \cdot (t+1) = 0 \vee (t-3) \cdot (t-2) = 0$
 $t = x^2 = 6 \vee x^2 = -1$ (k.n.) $\vee x^2 = 3 \vee x^2 = 2$
 $x = \pm\sqrt{6} \vee x = \pm\sqrt{3} \vee x = \pm\sqrt{2}.$

30a $\sqrt{2x-5} = 3$ (kwadrateren) $\Rightarrow 2x-5 = 9 \Rightarrow 2x = 14 \Rightarrow x = 7.$

30b $\sqrt{2x-5} = -3$ heeft geen oplossing, omdat een wortel niet negatief kan zijn.

■

31a ■ $x = \sqrt{5x+14}$ (kwadrateren)
 $x^2 = 5x+14$ $\begin{array}{|c|c|} \hline & \sqrt{5x+14} \\ \hline \end{array}$ 7
 $x^2 - 5x - 14 = 0$ $\begin{array}{|c|c|} \hline & \sqrt{5x-2+14} \\ \hline \end{array}$ 2
 $(x-7) \cdot (x+2) = 0$ ■
 $x = 7$ (voldoet) $\vee x = -2$ (voldoet niet).

31c ■ $5\sqrt{x} = x$ (kwadrateren)
 $25x = x^2$ $\begin{array}{|c|c|} \hline 5\sqrt{x}(0) & \\ \hline 5\sqrt{25} & 0 \\ \hline \end{array}$
 $0 = x^2 - 25x$ $\begin{array}{|c|c|} \hline 5\sqrt{x}(25) & \\ \hline 25 & \\ \hline \end{array}$
 $0 = x \cdot (x-25)$ ■
 $x = 0$ (voldoet) $\vee x = 25$ (voldoet).

32a ■ $4 - 3\sqrt{x} = 2$
 $-3\sqrt{x} = -2$
 $\sqrt{x} = \frac{2}{3}$ (kwadrateren) $\begin{array}{|c|c|} \hline 4-3\sqrt{4/9} & 2 \\ \hline \end{array}$
 $x = \frac{4}{9}$ (voldoet). ■

32c ■ $2x - 5\sqrt{x} = 3$
 $2x - 3 = 5\sqrt{x}$ (kwadrateren)
 $4x^2 - 6x - 6x + 9 = 25x$
 $4x^2 - 37x + 9 = 0$ ($a = 4$, $b = -37$ en $c = 9$)
 $D = (-37)^2 - 4 \cdot 4 \cdot 9 = 1225$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{37 \pm \sqrt{1225}}{2 \cdot 4} = \frac{37 \pm 35}{8}$
 $x = \frac{37+35}{8} = \frac{72}{8} = 9$ (voldoet) $\vee x = \frac{37-35}{8} = \frac{2}{8} = \frac{1}{4}$ (voldoet niet).

33a $2x + \sqrt{x} = 10$
 $\sqrt{x} = 10 - 2x$ (kwadrateren)
 $x = 100 - 20x - 20x + 4x^2$
 $0 = 4x^2 - 41x + 100$ ($a = 4$, $b = -41$ en $c = 100$)
 $D = (-41)^2 - 4 \cdot 4 \cdot 100 = 81$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{41 \pm \sqrt{81}}{2 \cdot 4} = \frac{41 \pm 9}{8}$
 $x = \frac{41+9}{8} = \frac{50}{8} = 6\frac{1}{4}$ (voldoet niet) $\vee x = \frac{41-9}{8} = \frac{32}{8} = 4$ (voldoet).

29b $|2x^3 - 5| = 15$
 $2x^3 - 5 = 15 \vee 2x^3 - 5 = -15$
 $2x^3 = 20 \vee 2x^3 = -10$
 $x^3 = 10 \vee x^3 = -5$
 $x = \sqrt[3]{10} \vee x = \sqrt[3]{-5}.$

29d $|x^6 - 10x^3| = 24$
 $x^6 - 10x^3 = 24 \vee x^6 - 10x^3 = -24$
 $x^6 - 10x^3 - 24 = 0 \vee x^6 - 10x^3 + 24 = 0$ (stel $x^3 = t$)
 $t^2 - 10t - 24 = 0 \vee t^2 - 10t + 24 = 0$
 $(t-12) \cdot (t+2) = 0 \vee (t-6) \cdot (t-4) = 0$
 $t = x^3 = 12 \vee x^3 = -2 \vee x^3 = 6 \vee x^3 = 4$
 $x = \sqrt[3]{12} \vee x = \sqrt[3]{-2} \vee x = \sqrt[3]{6} \vee x = \sqrt[3]{4}.$

3*2	9
Ans+5	14
Ans/2	7

(-8)^2-4*9*-20	784
J(784)	28
3*2	6
J(8*2+20)	6

3*2	6
J(8*2+20)	6
3*20/18	-3.33333333333
J(3*20/18)	6

31b ■ $3x = \sqrt{8x+20}$ (kwadrateren)
 $(3x)^2 = 8x+20$
 $9x^2 - 8x - 20 = 0$ ($a = 9$, $b = -8$ en $c = -20$)
 $D = (-8)^2 - 4 \cdot 9 \cdot -20 = 64 + 720 = 784$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{8 \pm \sqrt{784}}{2 \cdot 9} = \frac{8 \pm 28}{18}$
 $x = \frac{8+28}{18} = \frac{36}{18} = 2$ (voldoet) $\vee x = \frac{8-28}{18} = \frac{-20}{18} = -\dots$ (voldoet niet).

31c ■ $3x = \sqrt{18x+72}$ (kwadrateren)
 $(3x)^2 = 18x+72$
 $9x^2 - 18x - 72 = 0$
 $x^2 - 2x - 8 = 0$
 $(x-4) \cdot (x+2) = 0$
 $x = 4$ (voldoet) $\vee x = -2$ (voldoet niet).

3*4	12
J(18*4+72)	12
3*2	-6
J(18*-2+72)	6

32b ■ $5\sqrt{x} - 2x = 0$
 $5\sqrt{x} = 2x$ (kwadrateren)
 $25x = 4x^2$
 $0 = 4x^2 - 25x = 4x \cdot (x - \frac{25}{4})$
 $x = 0$ (voldoet) $x = 6\frac{1}{4}$ (voldoet). ■

5*J(0)-2*0	0
6+1/4+X	6.25
5J(X)-2X	0

32c ■ $5 - 2\sqrt{x} = 3$
 $2 = 2\sqrt{x}$
 $1 = \sqrt{x}$ (kwadrateren)
 $1 = x$ (voldoet). ■

5-3	2
1+X	1
5-2J(X)	3

33b $\sqrt{x+12} = x$ (kwadrateren)
 $x+12 = x^2$
 $0 = x^2 - x - 12$
 $0 = (x-4) \cdot (x+3)$
 $x = 4$ (voldoet) $\vee x = -3$ (voldoet niet).

4+X	4
J(X+12)	4
-3+X	-3
J(X+12)	3

- 33c $2x + \sqrt{x} = 6$
 $\sqrt{x} = 6 - 2x$ (kwadrateren)
 $x = 36 - 12x - 12x + 4x^2$
 $0 = 4x^2 - 25x + 36$ ($a = 4$, $b = -25$ en $c = 36$)
 $D = (-25)^2 - 4 \cdot 4 \cdot 36 = 49$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{25 \pm \sqrt{49}}{2 \cdot 4} = \frac{25 \pm 7}{8}$
 $x = \frac{25+7}{8} = \frac{32}{8} = 4$ (voldoet niet) $\vee x = \frac{25-7}{8} = \frac{18}{8} = 2\frac{1}{4}$ (voldoet).
- 33d $10 - x\sqrt{x} = 2$
 $8 = x\sqrt{x}$ (kwadrateren)
 $64 = x^2 \cdot x = x^3$
 $x = \sqrt[3]{64} = 4$ (voldoet).
- 34a $(x\sqrt{x})^2 + x\sqrt{x} - 6 = 0$ (stel $x\sqrt{x} = p$)
 $p^2 + p - 6 = 0$
 $(p+3) \cdot (p-2) = 0$
 $p = x\sqrt{x} = -3 \vee p = x\sqrt{x} = 2$
- 34b $x\sqrt{x} = -3$ ($x = 0$ voldoet niet, $x > 0$ of $x < 0$ kan ook niet)
 $x\sqrt{x} = 2$ (kwadrateren) $\Rightarrow x^2 \cdot x = 4 \Rightarrow x^3 = 4 \Rightarrow x = \sqrt[3]{4}$.
- 35a $x^3 - 9x\sqrt{x} + 8 = 0$ (stel $x\sqrt{x} = t$)
 $t^2 - 9t + 8 = 0$
 $(t-8) \cdot (t-1) = 0$
 $t = x\sqrt{x} = 8 \vee t = x\sqrt{x} = 1$ (kwadrateren)
 $x^2 \cdot x = x^3 = 8^2 = 64 \vee x^3 = 1^2 = 1$
 $x = \sqrt[3]{64} = 4 \vee x = \sqrt[3]{1} = 1$.
- 35b $x^3 + 27 = 28x\sqrt{x}$
 $x^3 - 28x\sqrt{x} + 27 = 0$ (stel $x\sqrt{x} = t$)
 $t^2 - 28t + 27 = 0$
 $(t-27) \cdot (t-1) = 0$
 $t = x\sqrt{x} = 27 \vee x\sqrt{x} = 1$ (kwadrateren)
 $x^3 = 27^2 = 729 \vee x^3 = 1^2 = 1$
 $x = \sqrt[3]{729} = 9 \vee x = \sqrt[3]{1} = 1$.
- 35c $8x^3 + 8 = 65x\sqrt{x}$
 $8x^3 - 65x\sqrt{x} + 8 = 0$ (stel $x\sqrt{x} = t$)
 $8t^2 - 65t + 8 = 0$ ($a = 8$, $b = -65$ en $c = 8$)
 $D = (-65)^2 - 4 \cdot 8 \cdot 8 = 3969$
 $t = \frac{-b \pm \sqrt{D}}{2a} = \frac{65 \pm \sqrt{3969}}{2 \cdot 8} = \frac{65 \pm 63}{16}$
 $x\sqrt{x} = \frac{65+63}{16} = \frac{128}{16} = 8 \vee x\sqrt{x} = \frac{65-63}{16} = \frac{2}{16} = \frac{1}{8}$ (kwadr.)
 $x^3 = 8^2 = 64 \vee x^3 = (\frac{1}{8})^2 = \frac{1}{64}$
 $x = \sqrt[3]{64} = 4 \vee x = \sqrt[3]{\frac{1}{64}} = \frac{1}{4}$.
- 35d $x^5 - 33x^2\sqrt{x} + 32 = 0$ (stel $x^2\sqrt{x} = t$)
 $t^2 - 33t + 32 = 0$
 $(t-32) \cdot (t-1) = 0$
 $t = x^2\sqrt{x} = 32 \vee x^2\sqrt{x} = 1$ (kwadrateren)
 $x^5 = 32^2 = 1024 \vee x^5 = 1^2 = 1$
 $x = \sqrt[5]{1024} = 4 \vee x = \sqrt[5]{1} = 1$.
- 36a $x^3 + 30 = 11x\sqrt{x}$
 $x^3 - 11x\sqrt{x} + 30 = 0$ ($x\sqrt{x} = t$)
 $t^2 - 11t + 30 = 0$
 $(t-6) \cdot (t-5) = 0$
 $t = x\sqrt{x} = 6 \vee x\sqrt{x} = 5$ (kwadrateren)
 $x^3 = 6^2 = 36 \vee x^3 = 5^2 = 25$
 $x = \sqrt[3]{36} \vee x = \sqrt[3]{25}$.
- 36b $x^3 + 125 = 126x\sqrt{x}$
 $x^3 - 126x\sqrt{x} + 125 = 0$ ($x\sqrt{x} = t$)
 $t^2 - 126t + 125 = 0$
 $(t-125) \cdot (t-1) = 0$
 $t = x\sqrt{x} = 125 \vee x\sqrt{x} = 1$ (kwadrateren)
 $x^3 = 125^2 = 15625 \vee x^3 = 1^2 = 1$
 $x = \sqrt[3]{15625} = 25 \vee x = \sqrt[3]{1} = 1$.
- 36c $x^5 + 10 = 7x^2\sqrt{x}$
 $x^5 - 7x^2\sqrt{x} + 10 = 0$ (stel $x^2\sqrt{x} = t$)
 $t^2 - 7t + 10 = 0$
 $(t-2) \cdot (t-5) = 0$
 $t = x^2\sqrt{x} = 2 \vee x^2\sqrt{x} = 5$ (kwadr.)
 $x^5 = 2^2 = 4 \vee x^5 = 5^2 = 25$
 $x = \sqrt[5]{4} \vee x = \sqrt[5]{25}$.
- 36d $32x^5 + 32 = 1025x^2\sqrt{x}$
 $32x^5 - 1025x^2\sqrt{x} + 32 = 0$ (stel $x^2\sqrt{x} = t$)
 $32t^2 - 1025t + 32 = 0$ ($a = 32$, $b = -1025$ en $c = 32$)
 $D = (-1025)^2 - 4 \cdot 32 \cdot 32 = 1046529$
 $t = \frac{-b \pm \sqrt{D}}{2a} = \frac{1025 \pm \sqrt{1046529}}{2 \cdot 32} = \frac{1025 \pm 1023}{64}$
 $x^2\sqrt{x} = \frac{1025+1023}{64} = 32 \vee x^2\sqrt{x} = \frac{1025-1023}{64} = \frac{1}{32}$ (kwadr.)
 $x^5 = 32^2 = 1024 \vee x^5 = (\frac{1}{32})^2 = \frac{1}{1024}$
 $x = \sqrt[5]{1024} = 4 \vee x = \sqrt[5]{\frac{1}{1024}} = \frac{1}{4}$.
- 37 $\bullet x - \sqrt{x} = 12 \Rightarrow x - 12 = \sqrt{x}$ (kwadrateren)
 $x^2 - 12x - 12x + 144 = x \Rightarrow x^2 - 25x + 144 = 0$
 $(x-9) \cdot (x-16) = 0 \Rightarrow x = 9$ (voldoet niet) $\vee x = 16$ (voldoet).
- $x - \sqrt{x} = 12 \Rightarrow x - \sqrt{x} - 12 = 0$ (stel $\sqrt{x} = t$)
 $t^2 - t - 12 = 0 \Rightarrow (t-4) \cdot (t+3) = 0$
 $t = \sqrt{x} = 4 \vee t = \sqrt{x} = -3$ (k.n.) $\Rightarrow x = 4^2 = 16$.

38a De kruisproducten bij de tabel geeft $x \cdot x = 2 \cdot (x + 4) \Rightarrow x^2 = 2x + 8 \Rightarrow x^2 - 2x - 8 = 0$.

38b $x^2 - 2x - 8 = 0 \Rightarrow (x - 4) \cdot (x + 2) = 0 \Rightarrow x = 4 \vee x = -2$.

39a $\frac{x+3}{x-1} = \frac{10}{x}$

$$x \cdot (x + 3) = 10 \cdot (x - 1)$$

$$x^2 + 3x = 10x - 10$$

$$x^2 - 7x + 10 = 0$$

$$(x - 5) \cdot (x - 2) = 0$$

$$x = 5 \text{ (vold.)} \vee x = 2 \text{ (vold.)}$$

39b $\frac{2x+3}{x+1} = \frac{2x+2}{x-1}$

$$(2x + 3) \cdot (x - 1) = (2x + 2) \cdot (x + 1)$$

$$2x^2 - 2x + 3x - 3 = 2x^2 + 2x + 2x + 2$$

$$-3x = 5$$

$$x = \frac{5}{-3} = -1\frac{2}{3} \text{ (vold.)}$$

39c $\frac{x-3}{x+1} = 1\frac{1}{2} = \frac{3}{2}$

$$2 \cdot (x - 3) = 3 \cdot (x + 1)$$

$$2x - 6 = 3x + 3$$

$$-x = 9$$

$$x = -9 \text{ (vold.)}$$

39d $\frac{x-1}{x} + 1 = 3$

$$\frac{x-1}{x} = 2 = \frac{2}{1}$$

$$2 \cdot x = 1 \cdot (x - 1)$$

$$2x = x - 1$$

$$x = -1 \text{ (vold.)}$$

VOLDOET NIET ALS EEN NOEMER NUL WORDT !!!

39e $\frac{3x+4}{x-1} = \frac{x+18}{x}$

$$x \cdot (3x + 4) = (x + 18) \cdot (x - 1)$$

$$3x^2 + 4x = x^2 - x + 18x - 18$$

$$2x^2 - 13x + 18 = 0 \text{ (a = 2, b = -13 en c = 18)}$$

$$D = (-13)^2 - 4 \cdot 2 \cdot 18 = 25$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{13 \pm \sqrt{25}}{2 \cdot 2} = \frac{13 \pm 5}{4} \quad \boxed{25}$$

$$x = \frac{13+5}{4} = \frac{18}{4} = 4,5 \text{ (vold.)} \vee x = \frac{13-5}{4} = \frac{8}{4} = 2 \text{ (vold.)}$$

39f $\frac{2x-5}{4-x} = \frac{x+2}{3x-4}$

$$(2x - 5) \cdot (3x - 4) = (x + 2) \cdot (4 - x)$$

$$6x^2 - 8x - 15x + 20 = 4x - x^2 + 8 - 2x$$

$$7x^2 - 25x + 12 = 0 \text{ (a = 7, b = -25 en c = 12)} \quad \boxed{(-25)^2 - 4 \cdot 7 \cdot 12} \quad 289$$

$$D = (-25)^2 - 4 \cdot 7 \cdot 12 = 289$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{25 \pm \sqrt{289}}{2 \cdot 7} = \frac{25 \pm 17}{14} \quad \boxed{42/14} \quad 17$$

$$x = \frac{25+17}{14} = \frac{42}{14} = 3 \text{ (vold.)} \vee x = \frac{25-17}{14} = \frac{8}{14} = \frac{4}{7} = 2 \text{ (vold.)} \quad 3$$

40a $\frac{5x^2-15}{x^2+5} = 0$

$$5x^2 - 15 = 0$$

$$5x^2 = 15$$

$$x^2 = 3$$

$$x = \sqrt{3} \vee x = -\sqrt{3} \quad (\text{vold.}) \quad (\text{vold.})$$

40b $\frac{x^2-3}{x^2+1} = \frac{x-1}{x^2+1}$

$$x^2 - 3 = x - 1$$

$$x^2 - x - 2 = 0$$

$$(x - 2) \cdot (x + 1) = 0$$

$$x = 2 \vee x = -1 \quad (\text{vold.}) \quad (\text{vold.})$$

40c $\frac{x^2-4}{2x+5} = \frac{x^2-4}{x+4}$

$$x^2 - 4 = 0 \vee 2x + 5 = x + 4$$

$$x^2 = 4 \vee x = -1$$

$$x = 2 \vee x = -2 \vee x = -1$$

$$(\text{vold.}) \quad (\text{vold.}) \quad (\text{vold.})$$

40d $\frac{x^2+1}{x+1} = \frac{x+3}{x+1}$

$$x^2 + 1 = x + 3$$

$$x^2 - x - 2 = 0$$

$$(x - 2) \cdot (x + 1) = 0$$

$$x = 2 \vee x = -1$$

$$(\text{vold.}) \quad (\text{vold. niet})$$

$\boxed{-1+1} \quad \emptyset$

41a $\frac{3x^2-10}{x^2+1} = 2 = \frac{2}{1}$

$$1 \cdot (3x^2 - 10) = 2 \cdot (x^2 + 1)$$

$$3x^2 - 10 = 2x^2 + 2$$

$$x^2 = 12$$

$$x = \sqrt{12} \vee x = -\sqrt{12}$$

$$(\text{vold.}) \quad (\text{vold.})$$

41b $\frac{x^3-8}{x^2+2} = \frac{x^3-8}{x+8}$

$$x^3 - 8 = 0 \vee x^2 + 2 = x + 8$$

$$x^3 = 8 \vee x^2 - x - 6 = 0$$

$$x = \sqrt[3]{8} = 2 \vee (x - 3) \cdot (x + 2) = 0 \quad \boxed{3 \cdot 48} \quad 2$$

$$x = 2 \vee x = 3 \vee x = -2$$

$$(\text{vold.}) \quad (\text{vold.}) \quad (\text{vold.})$$

41c $\frac{3x^2-10}{(x^2+1)^2} = \frac{2}{25}$

$$25 \cdot (3x^2 - 10) = 2 \cdot (x^2 + 1)^2$$

$$75x^2 - 250 = 2 \cdot (x^4 + x^2 + x^2 + 1)$$

$$75x^2 - 250 = 2x^4 + 4x^2 + 2$$

$$0 = 2x^4 - 71x^2 + 252 \text{ (stel } x^2 = t)$$

$$2t^2 - 71t + 252 = 0 \text{ (a = 2, b = -71 en c = 252)}$$

$$D = (-71)^2 - 4 \cdot 2 \cdot 252 = 3025$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{71 \pm \sqrt{3025}}{2 \cdot 2} = \frac{71 \pm 55}{4} \quad \boxed{3025} \quad 55$$

$$t = x^2 = \frac{71+55}{4} = 31,5 \vee x^2 = \frac{71-55}{4} = 4 \quad \boxed{(71+55)/4} \quad 31,5$$

$$x = \sqrt{31,5} \vee x = -\sqrt{31,5} \vee x = 2 \vee x = -2$$

$$(\text{vold.}) \quad (\text{vold.}) \quad (\text{vold.}) \quad (\text{vold.})$$

41d $\frac{6x^2-12}{(x^2-1)^2} = 1\frac{1}{3} = \frac{4}{3}$

$$3 \cdot (6x^2 - 12) = 4 \cdot (x^2 - 1)^2$$

$$18x^2 - 36 = 4 \cdot (x^4 - x^2 - x^2 + 1)$$

$$18x^2 - 36 = 4x^4 - 8x^2 + 4$$

$$0 = 4x^4 - 26x^2 + 40$$

$$0 = 2x^4 - 13x^2 + 20 \text{ (stel } x^2 = t)$$

$$2t^2 - 13t + 20 = 0 \text{ (a = 2, b = -13 en c = 20)}$$

$$D = (-13)^2 - 4 \cdot 2 \cdot 20 = 169 - 160 = 9 \quad \boxed{(-13)^2 - 4 \cdot 2 \cdot 20} \quad 9$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{13 \pm \sqrt{9}}{2 \cdot 2} = \frac{13 \pm 3}{4}$$

$$t = x^2 = \frac{13+3}{4} = 4 \vee x^2 = \frac{13-3}{4} = 2,5$$

$$x = 2 \vee x = -2 \vee x = \sqrt{2,5} \vee x = -\sqrt{2,5}$$

$$(\text{vold.}) \quad (\text{vold.}) \quad (\text{vold.}) \quad (\text{vold.})$$

42a $x = 0$ en $y = 3$ invullen in $x + 4y = 12$ geeft $0 + 4 \cdot 3 = 12$ (klopt).

42b $x = 4$ invullen in / geeft $y = -\frac{1}{4} \cdot 4 + 3 = -1 + 3 = 2 \Rightarrow (4, 2)$ ligt op /.

42c $y = -\frac{1}{4}x + 3 \Rightarrow 4y = -x + 12 \Rightarrow x + 4y = 12$.

43 $3x - y = 6$

$$\begin{array}{|c|c|c|} \hline x & 0 & 2 \\ \hline y & -6 & 0 \\ \hline \end{array}$$

$x + y = 1$

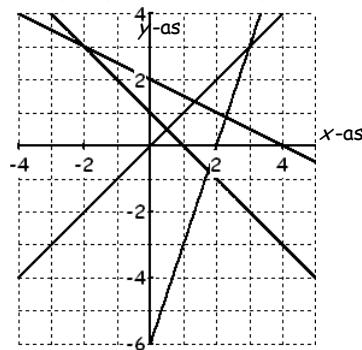
$$\begin{array}{|c|c|c|} \hline x & 0 & 1 \\ \hline y & 1 & 0 \\ \hline \end{array}$$

$x - y = 0$

$$\begin{array}{|c|c|c|} \hline x & 0 & 1 \\ \hline y & 0 & 1 \\ \hline \end{array}$$

$x + 2y = 4$

$$\begin{array}{|c|c|c|} \hline x & 4 & 0 \\ \hline y & 0 & 2 \\ \hline \end{array}$$



44a $y = 0 \Rightarrow 4x = 24 \Rightarrow x = 6$; dus snijpunt met de x -as: (6, 0).
 $x = 0 \Rightarrow -3y = 24 \Rightarrow y = -8$; dus snijpunt met de y -as: (0, -8).

44b $4 \cdot 8 - 3 \cdot 3 \neq 24 \Rightarrow (8, 3)$ ligt niet op l .
 $4 \cdot 18 - 3 \cdot 16 = 24 \Rightarrow (18, 16)$ ligt op l .

$4 \cdot -30 - 3 \cdot -48 = 24 \Rightarrow (-30, -48)$ ligt op l .

44c $4 \cdot 16 - 3 \cdot p = 24 \Rightarrow -3p = -40 \Rightarrow p = \frac{40}{3} = 13\frac{1}{3}$.

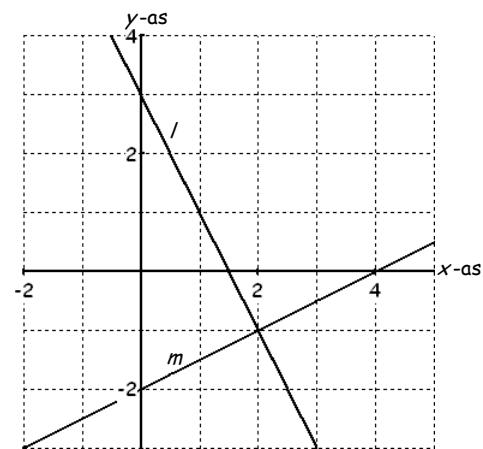
44d $4 \cdot q - 3 \cdot 48 = 24 \Rightarrow 4q = 168 \Rightarrow q = 42$.

$4 \cdot 8 - 3 \cdot 3$	23
$4 \cdot 18 - 3 \cdot 16$	24
$4 \cdot -30 - 3 \cdot -48$	24
$24 - 4 \cdot 16$	-40
Ans / -3	13.333333333
$24 + 3 \cdot 48$	168
Ans / 4	42

45a $\therefore 2x + y = 3$ gaat door (0, 3) en (1, 1);
 $m: x - 2y = 4$ gaat door (0, -2) en (4, 0).
(de grafieken van de lijnen in de figuur hiernaast)

45b Het snijpunt is (2, -1).

45c $x = 2 \wedge y = -1$ is zowel oplossing van $2x + y = 3$ als van $x - 2y = 4$.



46a $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 5x - 4y = -8 \\ -x + 4y = -12 \end{array} \right.} \\ \cancel{4x} = -20 \\ x = -5 \\ \cancel{-x + 4y = -12} \Rightarrow 5 + 4y = -12 \\ 4y = -17 \\ y = -\frac{17}{4} = -4\frac{1}{4}. \end{array}$

46b $\begin{array}{l} \cancel{\left\{ \begin{array}{l} -2x + y = 7 \\ -2x + 3y = -1 \end{array} \right.} \\ \cancel{-2y} = 8 \\ y = -4 \\ \cancel{-2x + y = 7} \Rightarrow -2x - 4 = 7 \\ -2x = 11 \\ x = -5\frac{1}{2}. \end{array}$

46c $\begin{array}{l} \cancel{\left\{ \begin{array}{l} -x - 3y = -8 \\ -2x + 3y = -1 \end{array} \right.} \\ \cancel{-3x} = -9 \\ x = 3 \\ \cancel{-2x + 3y = -1} \Rightarrow -6 + 3y = -1 \\ 3y = 5 \\ y = \frac{5}{3} = 1\frac{2}{3}. \end{array}$

47a $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 3x - 4y = 7 \\ 2x + 3y = 16 \end{array} \right.} \\ \cancel{5x - y} = 23 \\ \text{Nee, er is geen variabele geëlimineerd.} \end{array}$

47b $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 3x - 4y = 7 \\ 2x + 3y = 16 \end{array} \right.} \\ \cancel{x - 7y} = -9 \\ \text{Nee, er is geen variabele geëlimineerd.} \end{array}$

48a $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 3x + 5y = -7 \\ 2x + y = 0 \end{array} \right.} \mid 1 \\ \cancel{\left\{ \begin{array}{l} 3x + 5y = -7 \\ 10x + 5y = 0 \end{array} \right.} \\ \cancel{-7x} = -7 \\ x = 1 \\ \cancel{2x + y = 0} \Rightarrow 2 + y = 0 \\ y = -2. \end{array}$

48b $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 2x - 4y = 6 \\ 3x - y = 19 \end{array} \right.} \mid 1 \\ \cancel{\left\{ \begin{array}{l} 2x - 4y = 6 \\ 12x - 4y = 76 \end{array} \right.} \\ \cancel{-10x} = -70 \\ x = 7 \\ \cancel{3x - y = 19} \Rightarrow 21 - y = 19 \\ 2 = y. \end{array}$

48c $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 4x + y = 13 \\ x - 2y = 1 \end{array} \right.} \mid 2 \\ \cancel{\left\{ \begin{array}{l} 4x + y = 13 \\ 8x + 2y = 26 \end{array} \right.} \\ \cancel{x - 2y = 1} \\ \cancel{9x} = 27 \\ x = 3 \\ \cancel{4x + y = 13} \Rightarrow 12 + y = 13 \\ y = 1. \end{array}$

49a $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 5x + 2y = 69 \\ x + 3y = -7 \end{array} \right.} \mid 1 \\ \cancel{\left\{ \begin{array}{l} 5x + 2y = 69 \\ 5x + 15y = -35 \end{array} \right.} \\ \cancel{-13y} = 104 \\ y = -8 \\ \cancel{x + 3y = -7} \Rightarrow x - 24 = -7 \\ x = 17. \end{array}$

49b $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 2x - 5y = -19 \\ 5x + 4y = 35 \end{array} \right.} \mid 4 \\ \cancel{\left\{ \begin{array}{l} 2x - 5y = -19 \\ 25x + 20y = 175 \end{array} \right.} \\ \cancel{33x} = 99 \\ x = 3 \\ \cancel{5x + 4y = 35} \Rightarrow 15 + 4y = 35 \\ 4y = 20 \\ y = 5. \end{array}$

49c $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 0,8x + 0,2y = 1 \\ 0,3x - 0,3y = 1,5 \end{array} \right.} \mid 30 \\ \cancel{\left\{ \begin{array}{l} 24x + 6y = 30 \\ 6x - 6y = 30 \end{array} \right.} \\ \cancel{x - 2y = 1} \\ \cancel{30x} = 60 \\ x = 2 \\ \cancel{6x - 6y = 30} \Rightarrow 12 - 6y = 30 \\ -6y = 18 \\ y = -3. \end{array}$

50 $\begin{array}{l} \cancel{\left\{ \begin{array}{l} 3x - 2y = -12 \\ x + 4y = 38 \end{array} \right.} \mid 2 \\ \cancel{\left\{ \begin{array}{l} 6x - 4y = -24 \\ x + 4y = 38 \end{array} \right.} \\ \cancel{7x} = 14 \\ x = 2 \\ \cancel{x + 4y = 38} \Rightarrow 2 + 4y = 38 \\ 4y = 36 \\ y = 9. \end{array}$

51ab $x = 1 \wedge y = -2$ invullen:
 $-2 = 1^2 + b \cdot 1 + c$
 $-2 = 1 + b + c$
 $-3 = b + c.$
 $x = 2 \wedge y = 3$ invullen:
 $3 = 2^2 + b \cdot 2 + c$
 $3 = 4 + 2b + c$
 $-1 = 2b + c.$

51c $\begin{array}{l} \cancel{\left\{ \begin{array}{l} b + c = -3 \\ 2b + c = -1 \end{array} \right.} \\ \cancel{-b} = -2 \\ b = 2 \\ \cancel{b + c = -3} \Rightarrow 2 + c = -3 \\ c = -5. \end{array}$

52 (1, 8) op parabool $\Rightarrow 8 = a \cdot 1 + c$;
(2, 17) op parabool $\Rightarrow 17 = a \cdot 4 + c$.

$$\begin{aligned} &\triangleright \begin{cases} a+c=8 \\ 4a+c=17 \end{cases} \\ &\quad -3a = -9 \\ &\quad a=3 \\ &\triangleright \begin{cases} a+c=8 \\ c=5. \end{cases} \Rightarrow 3+c=8 \end{aligned}$$

53 (2, 8) op k $\Rightarrow 2a+b=8$;
(2, 8) op l $\Rightarrow 2b+a=8$.

$$\begin{aligned} &\triangleright \begin{cases} 2a+b=8 \\ a+2b=8 \end{cases} \\ &\quad \begin{array}{c|c} -2 & \\ \hline a+2b & 1 \\ -4a-2b & -16 \\ \hline a+2b & 8 \\ -3a & -8 \end{array} \\ &\quad a=\frac{8}{3} \\ &\triangleright \begin{cases} 2a+b=8 \\ a=\frac{8}{3} \end{cases} \Rightarrow \frac{16}{3}+b=8 \Rightarrow b=\frac{24}{3}-\frac{16}{3}=\frac{8}{3}=2\frac{2}{3}. \end{aligned}$$

54a (2, -1) op parabool $\Rightarrow -1 = 4 + 2p + q$;
(2, -1) op lijn $\Rightarrow -1 = 4p - q$.

$$\begin{aligned} &\triangleright \begin{cases} 2p+q=-5 \\ 4p-q=-1 \end{cases} \\ &\quad \begin{array}{c|c} + & \\ \hline 6p & -6 \\ p & -1 \end{array} \\ &\triangleright \begin{cases} 2p+q=-5 \\ q=-3. \end{cases} \Rightarrow -2+q=-5 \end{aligned}$$

54b De parabool $y = x^2 - x - 3$ snijden met de lijn $y = -2x + 3$.

$$\begin{aligned} &x^2 - x - 3 = -2x + 3 \\ &x^2 + x - 6 = 0 \\ &(x+3) \cdot (x-2) = 0 \\ &\begin{cases} x=-3 \\ y=-2 \cdot -3 + 3 = 9 \end{cases} \quad \begin{cases} x=2 \text{ (was gegeven)} \\ y=-2 \cdot 2 + 3 = -1. \end{cases} \end{aligned}$$

55 (-2, -10) op parabool $\Rightarrow -10 = a \cdot 4 + b \cdot -2 + c$ ①;
(0, 4) op parabool $\Rightarrow 4 = c$ ②;
(3, 5) op parabool $\Rightarrow 5 = a \cdot 9 + b \cdot 3 + c$ ③.
② invullen in ① en ③ geeft:
 $-10 = 4a - 2b + 4 \Rightarrow 4a - 2b = -14 \Rightarrow 2a - b = -7$ en
 $5 = 9a + 3b + 4 \Rightarrow 9a + 3b = 1.$

$$\begin{aligned} &\triangleright \begin{cases} 2a - b = -7 & |3 \\ 9a + 3b = 1 & |1 \end{cases} \\ &\quad \begin{array}{c|c} & \\ \hline 6a - 3b & -21 \\ 9a + 3b & 1 \\ \hline 15a & -20 \end{array} \\ &\quad a = -\frac{4}{3} \\ &\triangleright \begin{cases} 2a - b = -7 \\ a = -\frac{4}{3} \end{cases} \Rightarrow -\frac{8}{3} - b = -7 \\ &\quad -b = -\frac{21}{3} + \frac{8}{3} = -\frac{13}{3} \Rightarrow b = 4\frac{1}{3}. \end{aligned}$$

56 $\begin{cases} 2x + 3y = 12 \\ y = 4x - 10 \end{cases}$

$$\begin{aligned} &\triangleright \begin{cases} 2x + 3y = 12 & |1 \\ -4x + y = -10 & |-3 \end{cases} \\ &\quad \begin{array}{c|c} & \\ \hline 2x + 3y & 12 \\ 12x - 3y & 30 \end{array} \\ &\quad 14x = 42 \\ &\quad x = 3 \\ &\triangleright \begin{cases} x = 3 \\ y = 4x - 10 \end{cases} \Rightarrow y = 4 \cdot 3 - 10 = 2. \end{aligned}$$

57a $\begin{cases} 2x + 2y = 9 & ① \\ y = 4x - 3 & ② \end{cases}$
② in ① geeft: $2x + 2 \cdot (4x - 3) = 9$
 $2x + 8x - 6 = 9$
 $10x = 15$
 $x = 1,5$ in ②
 $y = 4 \cdot 1,5 - 3 = 3.$

57b $\begin{cases} y = \frac{1}{2}x + 1 & ① \\ 3x + 6y = 8 & ② \end{cases}$
① in ② geeft: $3x + 6 \cdot (\frac{1}{2}x + 1) = 8$
 $3x + 3x + 6 = 8$
 $6x = 2$
 $x = \frac{1}{3}$ in ①
 $y = \frac{1}{2} \cdot \frac{1}{3} + 1 = 1\frac{1}{6}.$

57c $\begin{cases} x = 5y - 3 & ① \\ 3x + 4y = 29 & ② \end{cases}$
① in ② geeft: $3 \cdot (5y - 3) + 4y = 29$
 $15y - 9 + 4y = 29$
 $19y = 38$
 $y = 2$ in ①
 $x = 5 \cdot 2 - 3 = 7.$

58a $\begin{cases} y = x^2 - 3 & ① \\ x - y = -3 & ② \end{cases}$
① in ② geeft: $x - (x^2 - 3) = -3$
 $x - x^2 + 3 = -3$
 $0 = x^2 - x - 6$
 $0 = (x-3) \cdot (x+2)$
 $\begin{cases} x = 3 \text{ in } ① \\ y = 9 - 3 = 6 \end{cases} \quad \begin{cases} x = -2 \text{ in } ① \\ y = 4 - 3 = 1. \end{cases}$

58b $\begin{cases} x^2 + y^2 = 25 \\ 3x + y = 5 \end{cases} \Rightarrow \begin{cases} x^2 + y^2 = 25 & ① \\ y = -3x + 5 & ② \end{cases}$
② in ① geeft: $x^2 + (-3x + 5)^2 = 25$
 $x^2 + 9x^2 - 15x - 15x + 25 = 25$
 $10x^2 - 30x = 0$
 $10x \cdot (x-3) = 0$
 $\begin{cases} x = 0 \text{ in } ② \\ y = 0 + 5 = 5 \end{cases} \quad \begin{cases} x = 3 \text{ in } ② \\ y = -9 + 5 = -4. \end{cases}$

58c
$$\begin{cases} x^2 + y^2 = 20 \\ xy = 8 \end{cases} \Rightarrow \begin{cases} x^2 + y^2 = 20 & \textcircled{1} \\ y = \frac{8}{x} & \textcircled{2} \end{cases}$$

$\textcircled{2}$ in $\textcircled{1}$ geeft: $x^2 + (\frac{8}{x})^2 = 20$
 $x^2 + \frac{64}{x^2} - 20 = 0$ (vermenigvuldigen met x^2)
 $x^4 - 20x^2 + 64 = 0$ (stel $x^2 = t$)
 $t^2 - 20t + 64 = 0$
 $(t - 16) \cdot (t - 4) = 0$
 $t = x^2 = 16 \vee t = x^2 = 4$

$\begin{cases} x = 4 \text{ in } \textcircled{2} \\ y = \frac{8}{4} = 2 \end{cases} \vee \begin{cases} x = -4 \text{ in } \textcircled{2} \\ y = \frac{8}{-4} = -2 \end{cases} \vee \begin{cases} x = 2 \text{ in } \textcircled{2} \\ y = \frac{8}{2} = 4 \end{cases} \vee \begin{cases} x = -2 \text{ in } \textcircled{2} \\ y = \frac{8}{-2} = -4. \end{cases}$

59 $x^4 - x^2 - 2 = 0$ kun je algebraïsch oplossen door $x^2 = t$ te stellen. Je krijgt $(t - 2) \cdot (t + 1) = 0$

$x^4 - x^3 - 2 = 0$ kun je niet algebraïsch oplossen.

$x^4 - x^3 - 2x = 0 \Rightarrow x \cdot (x^3 - x^2 - 2) = 0$ kun je niet algebraïsch oplossen.

$x^4 - x^3 - 2x^2 = 0 \Rightarrow x^2 \cdot (x^2 - x - 2) = 0 \Rightarrow x^2 \cdot (x - 2) \cdot (x + 1) = 0$ kun je algebraïsch oplossen.

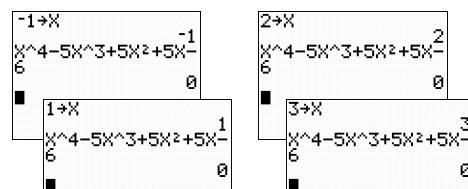
60a $x = -1, x = 1, x = 2$ en $x = 3$.

60b $x = -1$ geeft $y = (-1)^4 - 5 \cdot (-1)^3 + 5 \cdot (-1)^2 + 5 \cdot -1 - 6 = 0$ (klopt),

$x = 1$ geeft $y = 1^4 - 5 \cdot 1^3 + 5 \cdot 1^2 + 5 \cdot 1 - 6 = 0$ (klopt),

$x = 2$ geeft $y = 2^4 - 5 \cdot 2^3 + 5 \cdot 2^2 + 5 \cdot 2 - 6 = 0$ (klopt) en

$x = 3$ geeft $y = 3^4 - 5 \cdot 3^3 + 5 \cdot 3^2 + 5 \cdot 3 - 6 = 0$ (klopt).

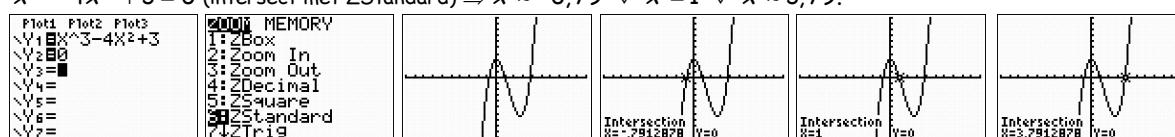


61a $x = -2, x = 2$ en $x = 4$.

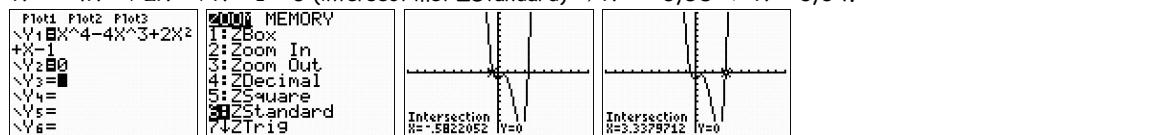
61b $x = -2 \vee x = 2 \vee x = 4$.

■ Neem GR-practicum 2 door. (uitwerkingen aan het eind)

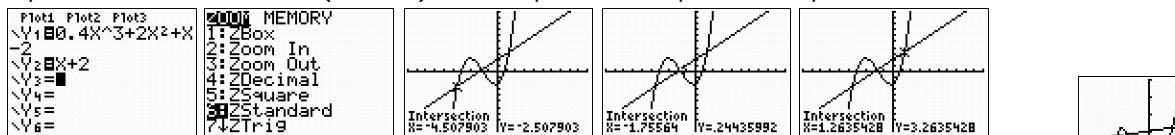
62a $x^3 - 4x^2 + 3 = 0$ (intersect met ZStandard) $\Rightarrow x \approx -0,79 \vee x = 1 \vee x \approx 3,79$.



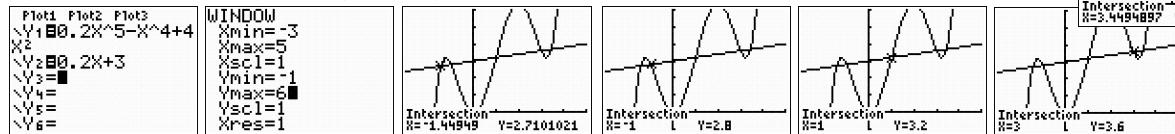
62b $x^4 - 4x^3 + 2x^2 + x - 1 = 0$ (intersect met ZStandard) $\Rightarrow x \approx -0,58 \vee x \approx 3,34$.



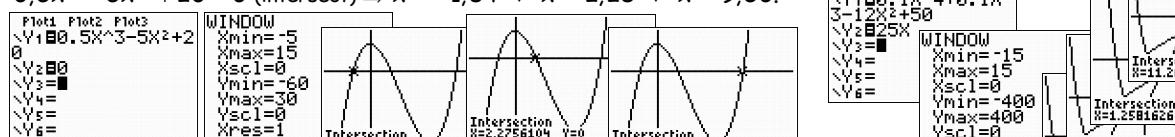
62c $0,4x^3 + 2x^2 + x - 2 = x + 2$ (intersect) $\Rightarrow x \approx -4,51 \vee x \approx -1,76 \vee x \approx 1,26$.



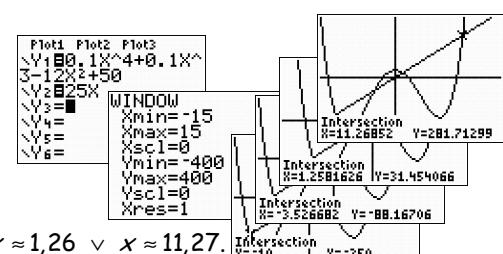
62d $0,2x^5 - x^4 + 4x^2 = 0,2x + 3$ (intersect) $\Rightarrow x \approx -1,45 \vee x = -1 \vee x = 1 \vee x = 3 \vee x \approx 3,45$.



63a $0,5x^3 - 5x^2 + 20 = 0$ (intersect) $\Rightarrow x \approx -1,84 \vee x \approx 2,28 \vee x \approx 9,56$.

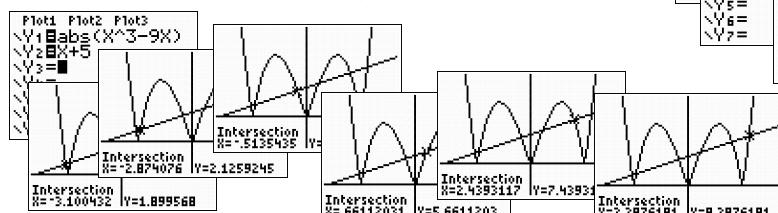


63b $0,1x^4 + 0,1x^3 - 12x^2 + 50 = 25x$ (intersect) $\Rightarrow x = -10 \vee x \approx -3,53 \vee x \approx 1,26 \vee x \approx 11,27$.

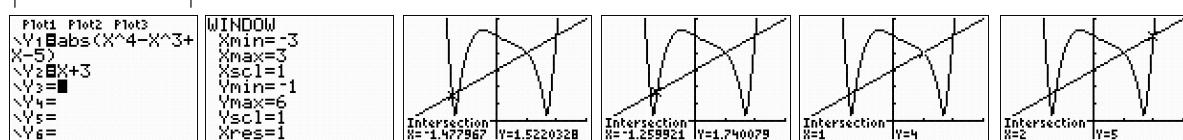


64a $|x^3 - 9x| = 5$ (intersect) $\Rightarrow x \approx -3,25 \vee x \approx -2,67 \vee x \approx -0,58 \vee x \approx 0,58 \vee x \approx 2,67 \vee x \approx 3,25.$

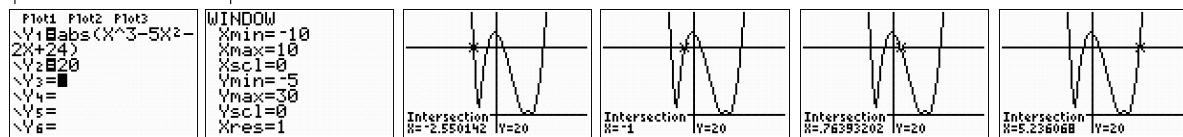
64b $|x^3 - 9x| = x + 5$ (intersect) $\Rightarrow x \approx -3,10 \vee x \approx -2,87 \vee x \approx -0,51 \vee x \approx 0,66 \vee x \approx 2,44 \vee x \approx 3,39.$



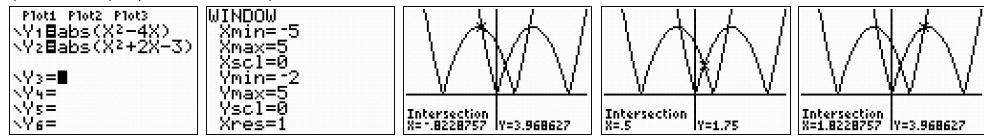
65a $|x^4 - x^3 + x - 5| = x + 3$ (intersect) $\Rightarrow x \approx -1,48 \vee x \approx -1,26 \vee x = 1 \vee x = 2.$



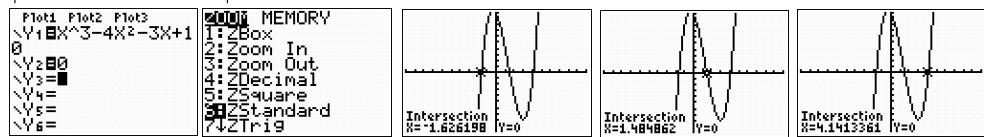
65b $|x^3 - 5x^2 - 2x + 24| = 20$ (intersect) $\Rightarrow x \approx -2,55 \vee x = -1 \vee x \approx 0,76 \vee x \approx 5,24.$



65c $|x^2 - 4x| = |x^2 + 2x - 3|$ (intersect) $\Rightarrow x \approx -0,82 \vee x = 0,5 \vee x \approx 1,82.$

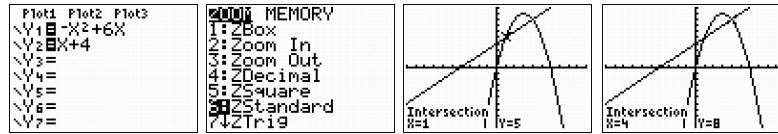


65d $|x^3 - 4x^2 - 3x + 10| = 0$ (intersect lukt niet) $\Rightarrow x^3 - 4x^2 - 3x + 10 = 0$ (intersect) $\Rightarrow x \approx -1,63 \vee x \approx 1,48 \vee x \approx 4,14.$



66a $-x^2 + 6x = x + 4$ (intersect) $\Rightarrow x = 1 \vee x = 4.$

66b Uit de plot volgt nu: $1 < x < 4$. (voor x tussen 1 en 4)



67a $x^2 - 3x = 14$ (intersect) $\Rightarrow x \approx -2,531 \vee x \approx 5,531.$

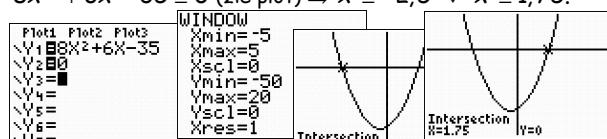
$x^2 - 3x \leq 14$ (zie plot) $\Rightarrow -2,531 \leq x \leq 5,531.$

67b $x^2 + 2x = 11$ (intersect) $\Rightarrow x \approx -4,464 \vee x \approx 2,464.$

$x^2 + 2x > 11$ (zie plot) $\Rightarrow x < -4,464 \vee x > 2,464.$

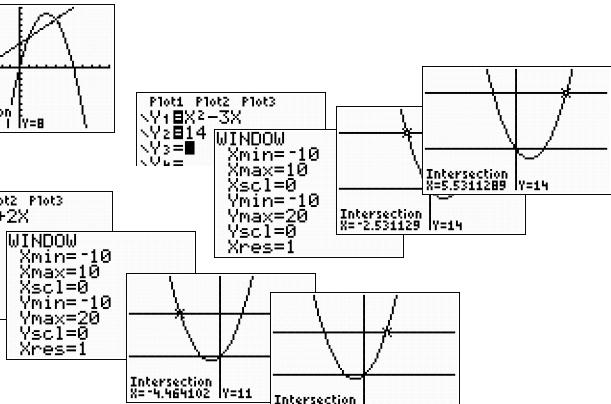
67c $8x^2 + 6x - 35 = 0$ (intersect) $\Rightarrow x = -2,5 \vee x = 1,75.$

$8x^2 + 6x - 35 \geq 0$ (zie plot) $\Rightarrow x \leq -2,5 \vee x \geq 1,75.$



67d $x^3 + 4,5x^2 = 19x + 60$ (intersect) $\Rightarrow x = -6 \vee x = -2,5 \vee x = 4.$

$x^3 + 4,5x^2 < 19x + 60$ (zie plot) $\Rightarrow x < -6 \vee -2,5 < x < 4.$



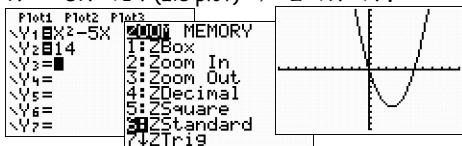
68a $x^2 - 5x = 14$ (niet met intersect !!!)

$$x^2 - 5x - 14 = 0$$

$$(x-7) \cdot (x+2) = 0$$

$$x = 7 \vee x = -2$$

$$x^2 - 5x < 14 \text{ (zie plot)} \Rightarrow -2 < x < 7.$$



68c $x^2 - 4x = -x^2 - 5x + 6$ (niet intersect)

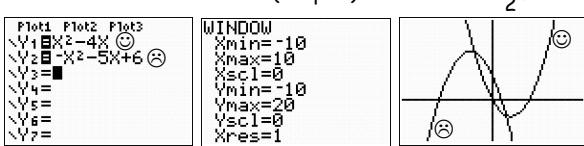
$$2x^2 + x - 6 = 0 \quad (a = 2, b = 1 \text{ en } c = -6)$$

$$D = 1^2 - 4 \cdot 2 \cdot -6 = 1 + 48 = 49$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{49}}{2 \cdot 2} = \frac{-1 \pm 7}{4}$$

$$x = \frac{-1+7}{4} = \frac{6}{4} = 1\frac{1}{2} \vee x = \frac{-1-7}{4} = \frac{-8}{4} = -2.$$

$$x^2 - 4x \leq -x^2 - 5x + 6 \text{ (zie plot)} \Rightarrow -2 \leq x \leq 1\frac{1}{2}.$$



69a $0,1x^3 - 2x^2 + 8x + 10 = -x + 15$ (intersect) $\Rightarrow x \approx 0,65 \vee x \approx 5,66 \vee x \approx 13,69.$

$$0,1x^3 - 2x^2 + 8x + 10 \geq -x + 15 \text{ (zie plot)} \Rightarrow 0,65 \leq x \leq 5,66 \vee x \geq 13,69.$$

69b $-0,5x^4 + 3x^3 - 4x^2 + 8 = x + 7$ (intersect)

$$x \approx -0,52 \vee x \approx 0,45 \vee x \approx 2,29 \vee x \approx 3,78.$$

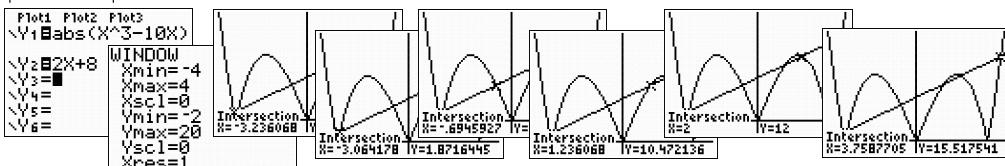
$$-0,5x^4 + 3x^3 - 4x^2 + 8 \geq x + 7 \text{ (zie plot)}$$

$$-0,52 \leq x \leq 0,45 \vee 2,29 \leq x \leq 3,78.$$

69c $|x^3 - 10x| = 2x + 8$ (intersect)

$$x \approx -3,24 \vee x \approx -3,06 \vee x \approx -0,69 \vee x \approx 1,24 \vee x = 2 \vee x \approx 3,76.$$

$$|x^3 - 10x| \leq 2x + 8 \text{ (zie plot)} \Rightarrow -3,24 \leq x \leq -3,06 \vee -0,69 \leq x \leq 1,24 \vee 2 \leq x \leq 3,76.$$



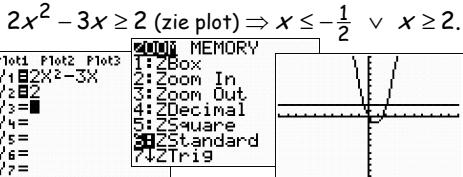
69d $|x^4 + x^2 - 5x - 10| = 8 - |2x - 4|$ (intersect)

$$x \approx -1,32 \vee x \approx -1,10 \vee x \approx 1,69 \vee x \approx 2,21.$$

$$|x^4 + x^2 - 5x - 10| \leq 8 - |2x - 4| \text{ (zie plot)}$$

$$-1,32 \leq x \leq -1,10 \vee 1,69 \leq x \leq 2,21.$$

68b $2x^2 - 3x - 2 = 0 \quad (a = 2, b = -3 \text{ en } c = -2)$
 $D = (-3)^2 - 4 \cdot 2 \cdot -2 = 9 + 16 = 25$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{3 \pm \sqrt{25}}{2 \cdot 2} = \frac{3 \pm 5}{4}$
 $x = \frac{3+5}{4} = \frac{8}{4} = 2 \vee x = \frac{3-5}{4} = \frac{-2}{4} = -\frac{1}{2}.$



68d $x^3 + 2x^2 = 3x$ (niet intersect)

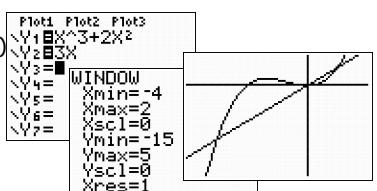
$$x^3 + 2x^2 - 3x = 0$$

$$x \cdot (x^2 + 2x - 3) = 0$$

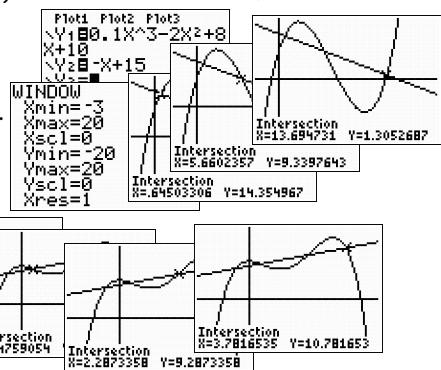
$$x \cdot (x+3) \cdot (x-1) = 0$$

$$x = 0 \vee x = -3 \vee x = 1.$$

$$x^3 + 2x^2 > 3x \text{ (zie plot)} \Rightarrow -3 < x < 0 \vee x > 1.$$



Plot1 Plot2 Plot3
Y1: 0.1x^3 - 2x^2 + 8
Y2: -x + 15
Y3: = WINDOW
Xmin=-10
Xmax=10
Ymin=-10
Ymax=20
Ysc1=0
Ysc2=0
Xres=1



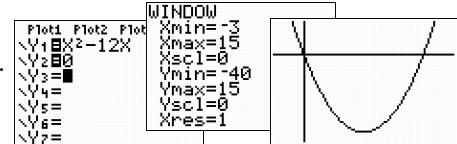
70a $x^2 + px + p = 0 \quad (a = 1, b = p \text{ en } c = p) \Rightarrow D = p^2 - 4 \cdot 1 \cdot p = p^2 - 4p.$

70b Twee oplossingen $\Rightarrow D = p^2 - 4p > 0.$

71a $x^2 + px + 3p = 0 \quad (a = 1, b = p \text{ en } c = 3p) \Rightarrow D = p^2 - 4 \cdot 1 \cdot 3p = p^2 - 12p.$

$$D = 0 \Rightarrow p^2 - 12p = p \cdot (p-12) = 0 \text{ (of intersect)} \Rightarrow p = 0 \vee p = 12.$$

Twee oplossingen $\Rightarrow D > 0$ (zie plot) $\Rightarrow p < 0 \vee p > 12.$



71b $px^2 + (p-4) \cdot x + 0,5 = 0 \quad (a = p \neq 0, b = p-4 \text{ en } c = 0,5) \Rightarrow D = (p-4)^2 - 4 \cdot p \cdot 0,5.$

$$D = 0 \Rightarrow p^2 - 10p + 16 = (p-8) \cdot (p-2) = 0 \text{ (of intersect)} \Rightarrow p = 8 \vee p = 2.$$

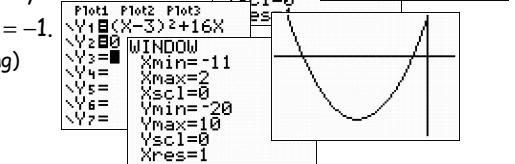
Twee oplossingen $\Rightarrow D > 0$ (zie plot) $\Rightarrow p < 0 \vee 0 < p < 2 \vee p > 8. (p = 0 \text{ geeft 1 oplossing})$

70b Twee oplossingen $\Rightarrow D = p^2 - 4p > 0.$

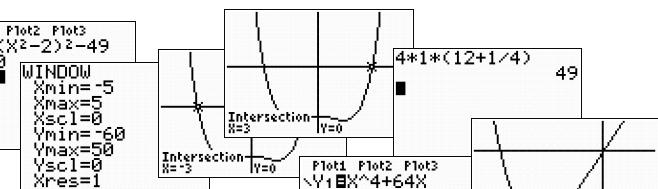
71c $px^2 + (p-3) \cdot x - 4 = 0 \quad (a = p \neq 0, b = p-3 \text{ en } c = -4) \Rightarrow D = (p-3)^2 - 4 \cdot p \cdot -4.$

$$D = 0 \Rightarrow p^2 + 10p + 9 = (p+9) \cdot (p+1) = 0 \text{ (of intersect)} \Rightarrow p = -9 \vee p = -1.$$

Geen oplossingen $\Rightarrow D < 0$ (zie plot) $\Rightarrow -9 < p < -1. (p = 0 \text{ geeft 1 oplossing})$



72a $x^2 + (p^2 - 2)x + 12\frac{1}{4} = 0$ ($a = 1$, $b = p^2 - 2$ en $c = 12\frac{1}{4}$).
 $D = (p^2 - 2)^2 - 4 \cdot 1 \cdot 12\frac{1}{4} = (p^2 - 2)^2 - 49.$
 $D = 0 \Rightarrow (p^2 - 2)^2 = 49 \Rightarrow p^2 - 2 = \pm 7 \Rightarrow p^2 = 2 \pm 7 \Rightarrow p^2 = 9 \vee p^2 = -5$ (kan niet) \Rightarrow (of niet intersect) $p = \pm 3.$
Twee oplossingen $\Rightarrow D > 0$ (zie plot) $\Rightarrow p < -3 \vee p > 3.$



72b $px^3 + p^2x^2 - 16x = x \cdot (px^2 + p^2x - 16) = 0 \Rightarrow x = 0 \vee px^2 + p^2x - 16 = 0.$

Drie oplossingen als $px^2 + p^2x - 16 = 0$ ($a = p \neq 0$, $b = p^2$ en $c = -16$) twee oplossingen heeft.

$$D = (p^2)^2 - 4 \cdot p \cdot -16 = p^4 + 64p.$$

$$D = 0 \Rightarrow p^4 + 64p = p \cdot (p^3 + 64) = 0 \Rightarrow p = 0 \vee p^3 = -64 \Rightarrow$$
 (of niet intersect) $p = 0 \vee p = \sqrt[3]{-64} = -4.$

Drie oplossingen $\Rightarrow D > 0$ (zie plot) $\Rightarrow p < -4 \vee p > 0.$

($p = 0$ geeft $-16x = 0 \Rightarrow x = 0 \Rightarrow 1$ oplossing)

$$\boxed{\begin{array}{l} 3 \sqrt{-64} \\ -4 \end{array}}$$

72c $px^3 + 2px^2 - 3x^2 + \frac{1}{4}x = px^3 + (2p - 3)x^2 + \frac{1}{4}x = x \cdot (px^2 + (2p - 3)x + \frac{1}{4}) = 0 \Rightarrow x = 0 \vee px^2 + (2p - 3)x + \frac{1}{4} = 0.$

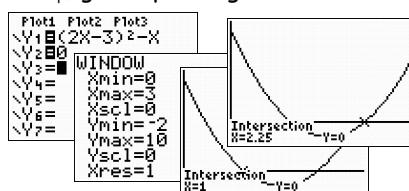
Eén oplossing als $px^2 + (2p - 3)x + \frac{1}{4} = 0$ ($a = p \neq 0$, $b = 2p - 3$ en $c = \frac{1}{4}$) geen oplossing heeft.

$$D = (2p - 3)^2 - 4 \cdot p \cdot \frac{1}{4} = (2p - 3)^2 - p.$$

$$D = 0 \Rightarrow (2p - 3)^2 - p = 0 \text{ (exact of intersect)} \Rightarrow p = 1 \vee p = 2\frac{1}{4}.$$

Eén oplossing $\Rightarrow D < 0$ (zie plot) $\Rightarrow 1 < p < 2\frac{1}{4}.$

($p = 0$ geeft $-3x^2 + \frac{1}{4}x = x \cdot (-3x + \frac{1}{4}) = 0 \Rightarrow 2$ oplossingen)



Diagnostische toets

- D1a $3x^2 - x = 0$ D1b $3x^2 - 9x = 12$ D1c $3x^2 - x = 2$
- $$x \cdot (3x - 1) = 0$$
- $$x = 0 \vee 3x - 1 = 0$$
- $$x = 0 \vee 3 = 1$$
- $$x = 0 \vee x = \frac{1}{3}.$$
- $$3x^2 - 9x - 12 = 0$$
- $$x^2 - 3x - 4 = 0$$
- $$(x - 4) \cdot (x + 1) = 0$$
- $$x = 4 \vee x = -1.$$
- $$3x^2 - x - 2 = 0 \quad (a = 3, b = -1 \text{ en } c = -2)$$
- $$D = (-1)^2 - 4 \cdot 3 \cdot -2 = 1 + 24 = 25$$
- $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{1 \pm \sqrt{25}}{2 \cdot 3} = \frac{1 \pm 5}{6}$$
- $$x = \frac{1+5}{6} = \frac{6}{6} = 1 \vee x = \frac{1-5}{6} = \frac{-4}{6} = -\frac{2}{3}.$$
- D1d $x^2 + 14 = 16$ D1e $(2x - 3)^2 = 81$ D1f $(3x + 2) \cdot (x - 1) = 0$
- $$x^2 = 2$$
- $$x = \pm\sqrt{2}.$$
- $$(x = \sqrt{2} \vee x = -\sqrt{2})$$
- $$2x - 3 = \pm\sqrt{81} = \pm 9$$
- $$2x = 3 \pm 9$$
- $$x = 1,5 \pm 4,5$$
- $$x = 6 \vee x = -3.$$
- $$3x + 2 = 0 \vee x - 1 = 0$$
- $$3x = -2 \vee x = 1$$
- $$x = -\frac{2}{3} \vee x = 1.$$
- D1g $x^2 = 7x + 13$ D1h $(3x + 2) \cdot (x - 1) = (x + 5) \cdot x$
- $$x^2 - 7x - 13 = 0 \quad (a = 1, b = -7 \text{ en } c = -13)$$
- $$D = (-7)^2 - 4 \cdot 1 \cdot -13 = 49 + 52 = 101$$
- $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{7 \pm \sqrt{101}}{2 \cdot 1} = \frac{7 \pm \sqrt{101}}{2}$$
- $$x = \frac{7 + \sqrt{101}}{2} \vee x = \frac{7 - \sqrt{101}}{2}.$$
- $$3x^2 - 3x + 2x - 2 = x^2 + 5x$$
- $$2x^2 - 6x - 2 = 0$$
- $$x^2 - 3x - 1 = 0 \quad (a = 1, b = -3 \text{ en } c = -1)$$
- $$D = (-3)^2 - 4 \cdot 1 \cdot -1 = 9 + 4 = 13$$
- $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{3 \pm \sqrt{13}}{2 \cdot 1} = \frac{3 \pm \sqrt{13}}{2}$$
- $$x = \frac{3 + \sqrt{13}}{2} \vee x = \frac{3 - \sqrt{13}}{2}.$$
- D1i $(x + 2) \cdot (x + 2) = 3x + 7$ D1j $(x - 3) \cdot (x - 3) - (x + 1) \cdot (x + 1) = (x - 4) \cdot (x - 4)$
- $$x^2 + 2x + 2x + 4 = 3x + 7$$
- $$x^2 + x - 3 = 0 \quad (a = 1, b = 1 \text{ en } c = -3)$$
- $$D = 1^2 - 4 \cdot 1 \cdot -3 = 1 + 12 = 13$$
- $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{13}}{2 \cdot 1} = \frac{-1 \pm \sqrt{13}}{2}$$
- $$x = \frac{-1 + \sqrt{13}}{2} \vee x = \frac{-1 - \sqrt{13}}{2}.$$
- $$x^2 - 3x - 3x + 9 - (x^2 + x + x + 1) = x^2 - 4x - 4x + 16$$
- $$x^2 - 6x + 9 - x^2 - 2x - 1 = x^2 - 8x + 16$$
- $$0 = x^2 + 8$$
- $$-8 = x^2 \quad (\text{kan niet})$$
- $$\text{geen oplossingen.}$$
- D2a $2x^2 + 4x + p = 0$ D2b $3x^2 + px + 27 = 0$ D2c $px^2 - 6x + 12 = 0$
- $$(a = 2, b = 4 \text{ en } c = p)$$
- $$D = 4^2 - 4 \cdot 2 \cdot p = 16 - 8p < 0$$
- $$-8p < -16$$
- $$p > \frac{-16}{-8} = 2.$$
- $$(a = 3, b = p \text{ en } c = 27)$$
- $$D = p^2 - 4 \cdot 3 \cdot 27 = p^2 - 324 > 0$$
- $$p^2 > 324 \quad \textcircled{O}$$
- $$p < -18 \vee p > 18.$$
- | | |
|----------------------------------------------|------------------|
| $\frac{4 \cdot 3 \cdot 27}{\text{Ans}(324)}$ | $\frac{324}{18}$ |
|----------------------------------------------|------------------|
- $$(a = p \neq 0, b = -6 \text{ en } c = 12)$$
- $$D = (-6)^2 - 4 \cdot p \cdot 12 = 36 - 48p = 0$$
- $$36 = 48p$$
- $$\frac{36}{48} = \frac{3}{4} = p;$$
- $$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{0}}{2 \cdot \frac{3}{4}} = \frac{6}{1,5} = \frac{12}{3} = 4;$$
- $$\text{voor } p = 0 \Rightarrow -6x + 12 = 0 \Rightarrow x = 2.$$
- D3a $x = 2 \Rightarrow 2^2 + 4 \cdot 2 + p = 0 \Rightarrow 4 + 8 + p = 0 \Rightarrow p = -12;$
 $x^2 + 4x - 12 = 0 \Rightarrow (x + 6) \cdot (x - 2) = 0 \Rightarrow x = -6 \vee x = 2 \text{ (was bekend).}$
- D3b $px^2 + 2x + 5 = 0 \quad (a = p \neq 0, b = 2 \text{ en } c = 5);$
 twee oplossingen $\Rightarrow D = 2^2 - 4 \cdot p \cdot 5 = 4 - 20p > 0 \Rightarrow -20p > -4 \Rightarrow p < \frac{-4}{-20} = \frac{1}{5}.$
 $(p=0 \text{ geeft een eerstegraadsvergelijking met één oplossing})$
- D4a $3x^3 + 5 = 86$ D4b $5x^4 - 6 = 9$ D4c $2x^3 + 19 = 5$
- $$3x^3 = 81$$
- | | |
|--------|----|
| 86-5 | 81 |
| 81/3 | |
| 27 | |
| 3 * 27 | 3 |
- $$x^3 = 27$$
- $$x = \sqrt[3]{27} = 3.$$
- $$5x^4 = 15$$
- | | |
|------|----|
| 9+6 | 15 |
| 15/5 | |
| 3 | |
- $$x^4 = 3$$
- | | |
|--------------|---|
| 4 * \sqrt{3} | 3 |
| 1.316074013 | |
- $$x = \pm \sqrt[4]{3}.$$
- $$2x^3 = -14$$
- | | |
|-------|-----|
| 5-19 | -14 |
| Ans/2 | |
- $$x^3 = -7$$
- | | |
|----------------|----|
| 3 * \sqrt{Ans} | -7 |
| -1.912931183 | |
- $$x = \sqrt[3]{-7}.$$
- D4d $\frac{1}{2}(x+2)^4 = \frac{1}{32}$ D4e $100 - (2x+1)^5 = 68$ D4f $(2x+4)^3 = 10$
- $$(x+2)^4 = \frac{1}{16}$$
- | | |
|--------------------------|------|
| 1/32/(1/2) * Frac | 1/16 |
| 4 * \sqrt{(1/16)} * Frac | 1/2 |
- $$x+2 = \pm \sqrt[4]{\frac{1}{16}} = \pm \frac{1}{2}$$
- $$x = -2 \pm \frac{1}{2}$$
- $$x = -1\frac{1}{2} \vee x = -2\frac{1}{2}.$$
- $$32 = (2x+1)^5$$
- | | |
|------------------|----|
| 100-68 | 32 |
| 5 * \sqrt[5]{32} | 2 |
| Ans-1 | |
| Ans/2 | 1 |
- $$2x+1 = \sqrt[5]{32} = 2$$
- $$2x = 1$$
- $$x = \frac{1}{2}.$$
- $$2x+4 = \sqrt[3]{10}$$
- | | |
|-------------------------------------|--------------|
| 3 * \sqrt[3]{10} | 2.15443469 |
| Ans-4 | |
| 2x = -4 + \sqrt[3]{10} | -1.84556531 |
| Ans/2 | |
| x = -2 + \frac{1}{2} * \sqrt[3]{10} | -0.922782655 |

D5a $x^4 - 6x^2 + 5 = 0$ (stel $x^2 = t$)
 $t^2 - 6t + 5 = 0$
 $(t-5) \cdot (t-1) = 0$
 $t = x^2 = 5 \vee t = x^2 = 1$
 $x = \pm\sqrt{5} \vee x = \pm\sqrt{1} = \pm 1.$

D5b $5x^4 - 6x^2 + 1 = 0$ (stel $x^2 = t$)
 $5t^2 - 6t + 1 = 0$ ($a = 5$, $b = -6$ en $c = 1$)
 $D = (-6)^2 - 4 \cdot 5 \cdot 1 = 36 - 20 = 16$
 $t = \frac{-b \pm \sqrt{D}}{2a} = \frac{6 \pm \sqrt{16}}{2 \cdot 5} = \frac{6 \pm 4}{10} \Rightarrow x^2 = \frac{6+4}{10} = 1 \vee x^2 = \frac{6-4}{10} = \frac{2}{10} = \frac{1}{5}$
 $x = \pm\sqrt{1} = \pm 1 \vee x = \pm\sqrt{\frac{1}{5}}.$

D5c $x^4 - 6x^3 + 5x^2 = 0$
 $x^2 \cdot (x^2 - 6x + 5) = 0$
 $x^2 \cdot (x-5) \cdot (x-1) = 0$
 $x = 0$ (dubbel) $\vee x = 5 \vee x = 1.$

D5d $x^3 + 6x^2 + 2x = 0$
 $x \cdot (x^2 + 6x + 2) = 0$
 $x = 0 \vee x^2 + 6x + 2 = 0$ ($a = 1$, $b = 6$ en $c = 2$)
 $D = 6^2 - 4 \cdot 1 \cdot 2 = 36 - 8 = 28 \Rightarrow x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-6 \pm \sqrt{28}}{2 \cdot 1} = \frac{-6 \pm \sqrt{28}}{2}$
 $x = 0 \vee x = \frac{-6 + \sqrt{28}}{2} \vee x = \frac{-6 - \sqrt{28}}{2}.$

D5e $3x^6 + 3 = 10x^3$
 $3x^6 - 10x^3 + 3 = 0$ (stel $x^3 = t$)
 $3t^2 - 10t + 3 = 0$ ($a = 3$, $b = -10$ en $c = 3$)
 $D = (-10)^2 - 4 \cdot 3 \cdot 3 = 100 - 36 = 64$
 $t = \frac{-b \pm \sqrt{D}}{2a} = \frac{10 \pm \sqrt{64}}{2 \cdot 3} = \frac{10 \pm 8}{6} \Rightarrow x^3 = \frac{10+8}{6} = 3 \vee x^3 = \frac{10-8}{6} = \frac{2}{6} = \frac{1}{3}$
 $x = \sqrt[3]{3} \vee x = \sqrt[3]{\frac{1}{3}}.$

D5f $x^8 + x^4 = 42$
 $x^8 + x^4 - 42 = 0$ (stel $x^4 = t$)
 $t^2 + t - 42 = 0$
 $(t+7) \cdot (t-6) = 0$
 $x^4 = -7$ (k.n.) $\vee x^4 = 6$
 $x = \pm\sqrt[4]{6}.$

D6a $|x^2 - 4| = 21$
 $x^2 - 4 = 21 \vee x^2 - 4 = -21$
 $x^2 = 25 \vee x^2 = -17$ (kan niet)
 $x = \pm\sqrt{25} = \pm 5.$

D6b $|4x^3 - 5| = 17$
 $4x^3 - 5 = 17 \vee 4x^3 - 5 = -17$
 $4x^3 = 22 \vee 4x^3 = -12$
 $x^3 = 5,5 \vee x^3 = -3$
 $x = \sqrt[3]{5,5} \vee x = \sqrt[3]{-3}.$

D7a $\sqrt{3x+5} + 1 = 5$
 $\sqrt{3x+5} = 4$ (kwadrateren)
 $3x+5 = 16$
 $3x = 11$
 $x = \frac{11}{3} = 3\frac{2}{3}$ (voldoet).

D7b $3x = 5\sqrt{x+4}$ (kwadrateren)
 $9x^2 = 25(x+4)$
 $9x^2 - 25x - 100 = 0$ ($a = 9$, $b = -25$ en $c = -100$)
 $D = (-25)^2 - 4 \cdot 9 \cdot -100 = 4225$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{25 \pm \sqrt{4225}}{2 \cdot 9} = \frac{25 \pm 65}{18}$
 $x = \frac{25+65}{18} = 5$ (voldoet) $\vee x = \frac{25-65}{18} = -2\frac{2}{9}$ (voldoet niet).

$$\begin{array}{r} (-25)^2 - 4 \cdot 9 \cdot -100 \\ 4225 \\ \hline \end{array}$$

$$\begin{array}{r} \sqrt{4225} \\ 65 \\ \hline \end{array}$$

$$\begin{array}{r} (25+65)/18 \\ 5 \\ \hline \end{array}$$

$$\begin{array}{r} 5+x \\ 3x-5\sqrt{x+4} \\ \hline 0 \\ \hline \end{array}$$

$$\begin{array}{r} -40/18+x \\ 2,222222222 \\ 3x-5\sqrt{x+4} \\ \hline -13.33333333 \\ \hline \end{array}$$

D7c $x = \sqrt{x} + 6$
 $x - 6 = \sqrt{x}$ (kwadrateren)
 $x^2 - 6x - 6x + 36 = x$
 $x^2 - 13x + 36 = 0$
 $(x-9) \cdot (x-4) = 0$
 $x = 9$ (voldoet) $\vee x = 4$ (voldoet niet).

D7d $2x + 3\sqrt{x} = 2$
 $3\sqrt{x} = 2 - 2x$ (kwadrateren)
 $9x = 4 - 4x - 4x + 4x^2$
 $0 = 4x^2 - 17x + 4$ ($a = 4$, $b = -17$ en $c = 4$)
 $D = (-17)^2 - 4 \cdot 4 \cdot 4 = 225$
 $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{17 \pm \sqrt{225}}{2 \cdot 4} = \frac{17 \pm 15}{8}$
 $x = \frac{17+15}{8} = \frac{32}{8} = 4$ (voldoet niet) $\vee x = \frac{17-15}{8} = \frac{2}{8} = \frac{1}{4}$ (voldoet).

$$\begin{array}{r} (-17)^2 - 4 \cdot 4 \cdot 4 \\ 225 \\ \hline \end{array}$$

$$\begin{array}{r} \sqrt{225} \\ 15 \\ \hline \end{array}$$

$$\begin{array}{r} (17+15)/8+x \\ 4 \\ \hline \end{array}$$

$$\begin{array}{r} 2x+3\sqrt{x} \\ 14 \\ \hline \end{array}$$

$$\begin{array}{r} (17-15)/8+x \\ 2 \\ \hline \end{array}$$

$$\begin{array}{r} 2x+3\sqrt{x} \\ .25 \\ \hline \end{array}$$

D8a $x^3 - 189 = 20x\sqrt{x}$
 $x^3 - 20x\sqrt{x} - 189 = 0$ ($x\sqrt{x} = t$)
 $t^2 - 20t - 189 = 0$
 $(t-27) \cdot (t+7) = 0$
 $t = x\sqrt{x} = 27 \vee t = x\sqrt{x} = -7$ (k.n.) (kwadrateren)
 $x^3 = 27^2 = 729$
 $x = \sqrt[3]{729} = 9$ (voldoet).

D8b $x^5 + 12 = 8x^2\sqrt{x}$
 $x^5 - 8x^2\sqrt{x} + 12 = 0$ (stel $x^2\sqrt{x} = t$)
 $t^2 - 8t + 12 = 0$
 $(t-6) \cdot (t-2) = 0$
 $t = x^2\sqrt{x} = 6 \vee x^2\sqrt{x} = 2$ (kwadrateren)
 $x^5 = 6^2 = 36 \vee x^5 = 2^2 = 4$
 $x = \sqrt[5]{36}$ (voldoet) $\vee x = \sqrt[5]{4}$ (voldoet).

$$\begin{array}{r} 5 \cdot \sqrt{36+x} \\ 2.047672511 \\ \hline \end{array}$$

$$\begin{array}{r} x^2\sqrt{x} \\ 6 \\ \hline \end{array}$$

$$\begin{array}{r} 5 \cdot \sqrt{4+x} \\ 1.319507911 \\ \hline \end{array}$$

$$\begin{array}{r} x^2\sqrt{x} \\ 2 \\ \hline \end{array}$$

D9a $\frac{6x-18}{x+1} = 0$ (teller = 0 en noemer ≠ 0) $\Rightarrow 6x-18=0 \Rightarrow 6x=18 \Rightarrow x=3$ (voldoet, want de noemer ≠ 0).

D9b $\frac{x^2 - 5x + 6}{2x + 4} = 0$
 $x^2 - 5x + 6 = 0$
 $(x - 3) \cdot (x - 2) = 0$
 $x = 3 \text{ (vold.)} \quad \vee \quad x = 2 \text{ (vold.)}$

D9c $\frac{3x-5}{x+1} = \frac{x+2}{x+1}$
 $3x-5 = x+2$
 $2x = 7$
 $x = 3,5$ (vold.).

$$\begin{aligned} \text{D9d } & \frac{x^2 - 4}{2x + 1} = \frac{x^2 - 4}{x - 4} \\ & x^2 - 4 = 0 \quad \vee \quad 2x + 1 = x - 4 \\ & x^2 = 4 \quad \vee \quad x = 5 \\ & x = 2 \text{ (void.)} \quad \vee \quad x = -2 \text{ (void.)} \quad \vee \quad x = -5 \text{ (void.)} \end{aligned}$$

D9e $\frac{2x-1}{x+1} = \frac{x+3}{x-4}$

$$(2x-1) \cdot (x-4) = (x+3) \cdot (x+1)$$

$$2x^2 - 8x - x + 4 = x^2 + x + 3x + 3$$

$$x^2 - 13x + 1 = 0 \quad (a = 1, b = -13 \text{ en } c = 1)$$

$$D = (-13)^2 - 4 \cdot 1 \cdot 1 = 169 - 4 = 165$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{13 \pm \sqrt{165}}{2 \cdot 1} = \frac{13 \pm \sqrt{165}}{2}$$

$$x = \frac{13 + \sqrt{165}}{2} \quad (\text{vold.}) \quad \vee \quad x = \frac{13 - \sqrt{165}}{2} \quad (\text{vold.})$$

D9f $\frac{2x^2 - 4}{x+5} = 1\frac{3}{4} = \frac{7}{4}$
 $4 \cdot (2x^2 - 4) = 7 \cdot (x + 5)$
 $8x^2 - 16 = 7x + 35$
 $8x^2 - 7x - 51 = 0 \quad (a = 8, b = -7 \text{ en } c = -51)$
 $D = (-7)^2 - 4 \cdot 8 \cdot -51 = 1681$ $(-7)^2 - 4 \cdot 8 \cdot -51$
 $X = \frac{-b \pm \sqrt{D}}{2a} = \frac{7 \pm \sqrt{1681}}{2 \cdot 8} = \frac{7 \pm 41}{16}$ ■
 $X = \frac{7+41}{16} = \frac{48}{16} = 3 \text{ (vold.)} \quad \vee \quad X = \frac{7-41}{16} = \frac{-34}{16} = -2\frac{1}{8} \text{ (vold.)}$

$$\text{D10a} \Rightarrow \begin{cases} 4x + 5y = 27 & |1 \\ -2x + 3y = 25 & |2 \end{cases}$$

$$\begin{cases} 4x + 5y = 27 \\ -4x + 6y = 50 \end{cases} +$$

$$11y = 77$$

$$y = 7$$

$$\begin{cases} 4x + 5y = 27 \\ 4x = -8 \\ x = -2. \end{cases} \Rightarrow 4x + 35 = 27$$

$$\text{D10b} \rightarrow \begin{cases} 2x + 3y = 7 & |2 \\ 5x - 2y = 8 & |3 \\ 4x + 6y = 14 \\ \hline 15x - 6y = 24 & + \\ 19x & = 38 \\ x = 2 & \\ \hline 2x + 3y = 7 & \end{cases} \Rightarrow \begin{cases} 4 + 3y \\ 3y = 3 \\ y = 1. \end{cases}$$

$$\begin{aligned} \text{D11} \square (2, 18) \text{ op parabol} &\Rightarrow 18 = a \cdot 4 + b \cdot 2; \\ (-4, 0) \text{ op parabol} &\Rightarrow 0 = a \cdot 16 + b \cdot -4. \\ \rightarrow \left\{ \begin{array}{l} 4a + 2b = 18 \\ \frac{8a - 2b = 0}{12a} + \\ 18 \end{array} \right. \\ \left. \begin{array}{l} a = 1\frac{1}{2} \\ 4a + 2b = 18 \end{array} \right\} &\Rightarrow 6 + 2b = 18 \\ 2b &= 12 \\ b &= 6. \end{aligned}$$

D12a
$$\begin{cases} 5x - 3y = 3 & \textcircled{1} \\ y = \frac{2}{3}x - 4 & \textcircled{2} \end{cases}$$

 $\textcircled{2}$ in $\textcircled{1}$ geeft: $5x - 3 \cdot \left(\frac{2}{3}x - 4\right) = 3$

$$\text{D12b} \quad \begin{cases} 2x + 3y = 10 & \textcircled{1} \\ y = x^2 - 4x + 6 & \textcircled{2} \end{cases}$$

$\textcircled{2}$ in $\textcircled{1}$ geeft: $2x + 3 \cdot (x^2 - 4x + 6) = 10$

$$2x + 3x^2 - 12x + 18 = 10$$

$$3x^2 - 10x + 8 = 0 \quad (a = 3, b = -10 \text{ en } c = 8)$$

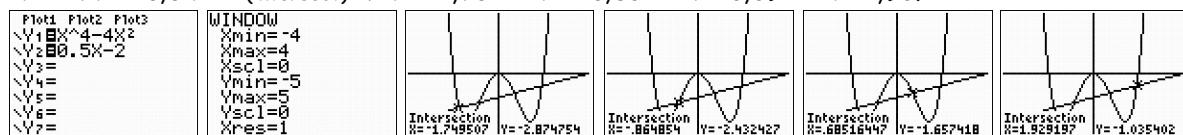
$$D = (-10)^2 - 4 \cdot 3 \cdot 8 = 4 \quad \boxed{(-10)^2 - 4 \cdot 3 \cdot 8} \quad 4$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{10 \pm \sqrt{4}}{2 \cdot 3} = \frac{10 \pm 2}{6} \quad \blacksquare$$

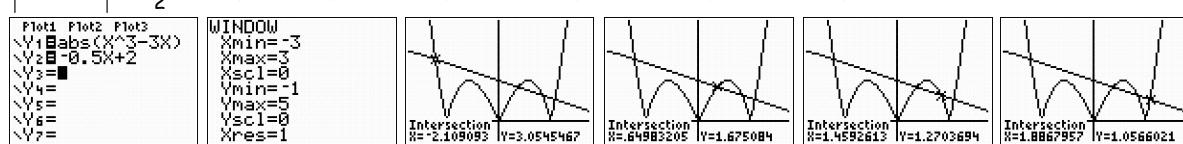
$$x = \frac{10+2}{6} = \frac{12}{6} = 2 \quad \vee \quad x = \frac{10-2}{6} = \frac{8}{6} = \frac{4}{3}$$

$$\begin{cases} x = 2 \text{ in ②} \\ y = 4 - 8 + 6 = 2 \end{cases} \quad \vee \quad \begin{cases} x = 1\frac{1}{3} \text{ in ②} \\ y = \frac{22}{9} = 2\frac{4}{9} \end{cases}$$

$$\text{D13a} \quad x^4 - 4x^2 = 0,5x - 2 \text{ (intersect)} \Rightarrow x \approx -1,75 \vee x \approx -0,86 \vee x \approx 0,69 \vee x \approx 1,93.$$

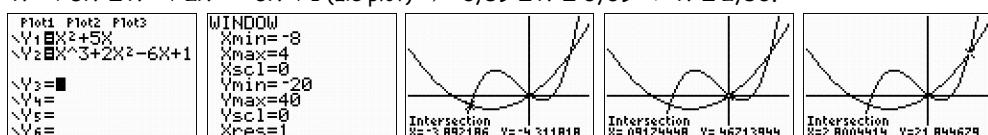


$$\text{D13b} \quad |x^3 - 3x| = -\frac{1}{2}x + 2 \text{ (intersect)} \Rightarrow x \approx -2,11 \vee x \approx 0,65 \vee x \approx 1,46 \vee x \approx 1,89.$$

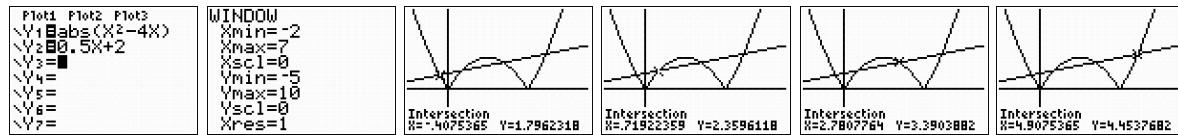


$$\text{D14a} \quad x^2 + 5x = x^3 + 2x^2 - 6x + 1 \text{ (intersect)} \Rightarrow x \approx -3,89 \vee x \approx 0,09 \vee x \approx 2,80.$$

$$x^2 + 5x \leq x^3 + 2x^2 - 6x + 1 \text{ (zie plot)} \Rightarrow -3,89 \leq x \leq 0,09 \quad \vee \quad x \geq 2,80.$$



D14b $\boxed{|x^2 - 4x| = \frac{1}{2}x + 2 \text{ (intersect)} \Rightarrow x \approx -0,41 \vee x \approx 0,72 \vee x \approx 2,78 \vee x \approx 4,91.}$
 $|x^2 - 4x| > \frac{1}{2}x + 2 \text{ (zie plot)} \Rightarrow x < -0,41 \vee 0,72 < x < 2,78 \vee x > 4,91.$



D15a $\boxed{3x^2 + 2x = 33 \text{ (niet met intersect !!!)}}$

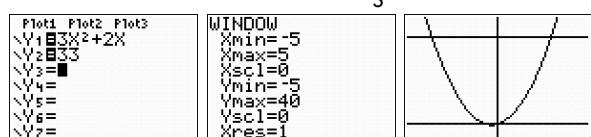
$$3x^2 + 2x - 33 = 0 \quad (a = 3, b = 2 \text{ en } c = -33)$$

$$D = 2^2 - 4 \cdot 3 \cdot -33 = 400$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-2 \pm \sqrt{400}}{2 \cdot 3} = \frac{-2 \pm 20}{6}$$

$$x = \frac{-2 + 20}{6} = \frac{18}{6} = 3 \quad \vee \quad x = \frac{-2 - 20}{6} = \frac{-22}{6} = -\frac{11}{3} = -3\frac{2}{3}.$$

$$3x^2 + 2x \geq 33 \text{ (zie plot)} \Rightarrow x \leq -3\frac{2}{3} \vee x \geq 3.$$



D16a $\boxed{px^2 + px - 2x + 4p = px^2 + (p-2)x + 4p = 0 \quad (a = p \neq 0, b = p-2 \text{ en } c = 4p)}$

$$D = (p-2)^2 - 4 \cdot p \cdot 4p = (p-2)^2 - 16p^2.$$

$$D = 0 \Rightarrow (p-2)^2 - 16p^2 = 0 \text{ (intersect)} \Rightarrow p = -\frac{2}{3} \vee p = 0,4.$$

Geen oplossing $\Rightarrow D < 0$ (zie plot) $\Rightarrow p < -\frac{2}{3} \vee p > 0,4.$
 $(p = 0 \text{ geeft } -2x = 0 \Rightarrow x = 0 \Rightarrow 1 \text{ oplossing})$

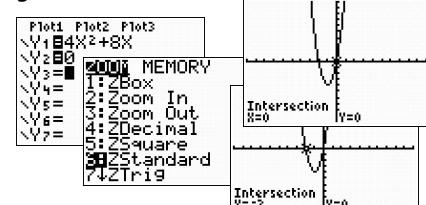
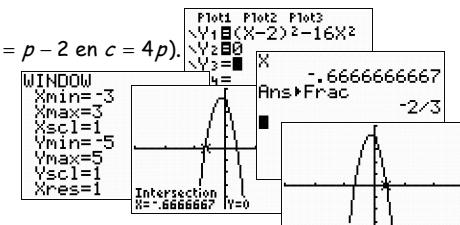
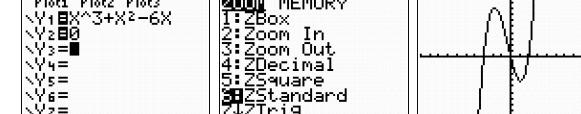
D16b $\boxed{px^3 + 2px^2 - 2x = x \cdot (px^2 + 2px - 2) = 0 \Rightarrow x = 0 \vee px^2 + 2px - 2 = 0.}$

Eén oplossing als $px^2 + 2px - 2 = 0 \quad (a = p \neq 0, b = 2p \text{ en } c = -2)$ geen oplossing heeft.

$$D = (2p)^2 - 4 \cdot p \cdot -2 = 4p^2 + 8p.$$

$$D = 0 \Rightarrow 4p^2 + 8p = 0 \Rightarrow 4p \cdot (p+2) = 0 \text{ (of intersect)} \Rightarrow p = 0 \vee p = -2.$$

$D < 0$ (zie plot) $\Rightarrow -2 < p < 0.$ ($p = 0$ geeft $-2x = 0 \Rightarrow x = 0 \Rightarrow$ ook 1 oplossing)
 Dus één oplossing als $-2 < p \leq 0.$



Gemengde opgaven 1. Vergelijkingen en ongelijkheden

- G1a $7x^2 = 5x$ G1b $2x^2 + x = 3$ G1c $(x+2) \cdot (x-6) = 9$
 $7x^2 - 5x = 0$ $2x^2 + x - 3 = 0$ ($a = 2$, $b = 1$ en $c = -3$) $x^2 - 6x + 2x - 12 = 9$
 $x \cdot (7x-5) = 0$ $D = 1^2 - 4 \cdot 2 \cdot -3 = 1 + 24 = 25$ $x^2 - 4x - 21 = 0$
 $x = 0 \vee 7x = 5$ $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{25}}{2 \cdot 2} = \frac{-1 \pm 5}{4}$ $(x-7) \cdot (x+3) = 0$
 $x = 0 \vee x = \frac{5}{7}$. $x = \frac{-1+5}{4} = \frac{4}{4} = 1 \vee x = \frac{-1-5}{4} = \frac{-6}{4} = -1\frac{1}{2}$. $x = 7 \vee x = -3$.
- G1d $(x-3)^2 - (x+1)^2 = x^2 - 1$ G1e $(2x-3)^2 = 36$ G1f $4 - (x-2)^2 = 7x - 3$
 $x^2 - 3x - 3x + 9 - (x^2 + x + x + 1) = x^2 - 1$ $2x - 3 = \pm 6$ $4 - (x^2 - 2x - 2x + 4) = 7x - 3$
 $x^2 - 6x + 9 - x^2 - 2x - 1 = x^2 - 1$ $2x = 3 \pm 6$ $4 - x^2 + 4x - 4 = 7x - 3$
 $0 = x^2 + 8x - 9$ $x = \frac{3+6}{2} \vee x = \frac{3-6}{2}$. $0 = x^2 + 3x - 3$ ($a = 1$, $b = 3$ en $c = -3$)
 $(x+9) \cdot (x-1) = 0$ $x = 4\frac{1}{2} \vee x = -1\frac{1}{2}$. $D = 3^2 - 4 \cdot 1 \cdot -3 = 9 + 12 = 21$
 $x = -9 \vee x = 1$. $x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-3 \pm \sqrt{21}}{2 \cdot 1} = \frac{-3 \pm \sqrt{21}}{2}$
 $x = \frac{-3 + \sqrt{21}}{2} \vee x = \frac{-3 - \sqrt{21}}{2}$.
- G2a $px^2 + 6x + 3p = 0$ ($a = p \neq 0$, $b = 6$ en $c = 3p$) Plot 1: $y_1 = 36 - 12x^2$
 $D = 6^2 - 4 \cdot p \cdot 3p = 36 - 12p^2$. Plot 2: $y_2 = 0$
 $D = 0 \Rightarrow 36 - 12p^2 = 0 \Rightarrow 36 = 12p^2 \Rightarrow p^2 = 3 \Rightarrow p = \pm\sqrt{3}$. Plot 3: $y_3 = 0$
 $D > 0$ (zie plot) $\Rightarrow -\sqrt{3} < p < \sqrt{3}$. $y_4 = 0$
 $(p = 0 \Rightarrow 6x = 0 \Rightarrow x = 0 \Rightarrow 1$ oplossing)
- G2b $6^2 + p \cdot 6 - 6p^2 = 0$ $p = 3$ geeft:
 $-6p^2 + 6p + 36 = 0$ $x^2 + 3x - 54 = 0$
 $p^2 - p - 6 = 0$ $(x-6) \cdot (x+9) = 0$
 $(p-3) \cdot (p+2) = 0$ $x = 6$ (bekend) $\vee x = -9$. $p = -2$ geeft:
 $p = 3 \vee p = -2$. $x^2 - 2x - 24 = 0$
 $x^2 - 2x + 1 = 0$
 $(x-1) \cdot (x-1) = 0 \Rightarrow x = 1$ (dubbel).
- G2c $px^2 - 2px + 4 = 0$ ($a = p \neq 0$, $b = -2p$ en $c = 4$) $p = -2$ geeft:
 $D = (-2p)^2 - 4 \cdot p \cdot 4 = 4p^2 - 16p$. $x^2 - 2x - 24 = 0$
 $D = 0 \Rightarrow 4p^2 - 16p = 0 \Rightarrow 4p \cdot (p-4) = 0 \Rightarrow p = 0 \vee p = 4$. $(x-6) \cdot (x+4) = 0$
 $(p = 0 \Rightarrow 4 = 0$ (kan niet) \Rightarrow geen oplossing)
 $p = 4 \Rightarrow 4x^2 - 8x + 4 = 0$
 $x^2 - 2x + 1 = 0$
 $(x-1) \cdot (x-1) = 0 \Rightarrow x = 1$ (dubbel).
- G3a $x^6 - 6x^3 + 5 = 0$ (stel $x^3 = t$) G3b $|x^4 - 7x^2| = 18$
 $t^2 - 6t + 5 = 0$
 $(t-5) \cdot (t-1) = 0$
 $t = x^3 = 5 \vee x^3 = 1$
 $x = \sqrt[3]{5} \vee x = \sqrt[3]{1} = 1$. $x^4 - 7x^2 = 18 \vee x^4 - 7x^2 = -18$ (stel $x^2 = t$)
 $t^2 - 7t - 18 = 0 \vee t^2 - 7t + 18 = 0$ ($a = 1$, $b = -7$ en $c = 18$)
 $(t-9) \cdot (t+2) = 0 \vee D = (-7)^2 - 4 \cdot 1 \cdot 18 < 0$ (geen oplossingen)
 $x^2 = 9 \vee x^2 = -2$ (k.n.) $\boxed{(-7)^2 - 4 \cdot 1 \cdot 18 = -23}$
 $x = 3 \vee x = -3$.
- G3c $10x^4 = 17x^2 + 657$ G3d $10 - (2x-1)^4 = 8$
 $10x^4 - 17x^2 - 657 = 0$ (stel $x^2 = t$) $2 = (2x-1)^4$
 $10t^2 - 17t - 657 = 0$ ($a = 10$, $b = -17$ en $c = -657$) $2x-1 = \pm\sqrt[4]{2}$
 $D = (-17)^2 - 4 \cdot 10 \cdot -657 = 26569$ $\boxed{\begin{array}{r} (-17)^2 - 4 \cdot 10 \cdot -657 \\ \hline 26569 \end{array}}$ $2x = 1 \pm \sqrt[4]{2}$
 $t = \frac{-b \pm \sqrt{D}}{2a} = \frac{17 \pm \sqrt{26569}}{2 \cdot 10} = \frac{17 \pm 163}{20}$ $x = \frac{1}{2} + \frac{1}{2}\sqrt[4]{2} \vee x = \frac{1}{2} - \frac{1}{2}\sqrt[4]{2}$
 $x^2 = \frac{17+163}{20} = \frac{180}{20} = 9 \vee x^2 = \frac{17-163}{20} = \frac{-147}{20} = -... \text{ (k.n.)} \Rightarrow x = 3 \vee x = -3$.
- G3e $x^5 - 16x^3 + 28x = 0$ G3f $x^3 - 3x\sqrt{x} - 108 = 0$ (stel $x\sqrt{x} = t$)
 $x \cdot (x^4 - 16x^2 + 28) = 0$ (stel $x^2 = t$) $t^2 - 3t - 108 = 0$
 $x = 0 \vee t^2 - 16t + 28 = 0$
 $x = 0 \vee (t-14) \cdot (t-2) = 0$
 $x = 0 \vee x^2 = 14 \vee x^2 = 2$
 $x = 0 \vee x = \pm\sqrt{14} \vee x = \pm\sqrt{2}$. $(t-12) \cdot (t+9) = 0$
 $x\sqrt{x} = 12$ (kwadrateren) $\vee x\sqrt{x} = -9$ (k.n.)
 $x^2 \cdot \sqrt{x} = x^3 = 144$ $\boxed{\begin{array}{r} 12^2 \\ 3 \cdot 144 \rightarrow x \\ \hline 144 \end{array}}$
 $x = \sqrt[3]{144}$ (voldoet). $x\sqrt{x} = 5 \cdot 241482788$
 $\boxed{\begin{array}{r} 5 \cdot 241482788 \\ \hline 12 \end{array}}$

G3g \square $6x^5 + 10x^2 \cdot \sqrt{x} - 464 = 0$ (stel $x^2 \cdot \sqrt{x} = t$)
 $6t^2 + 10t - 464 = 0$

$$3t^2 + 5t - 232 = 0 \quad (a=3, b=5 \text{ en } c=-232)$$

$$D = 5^2 - 4 \cdot 3 \cdot -232 = 2809$$

$$t = \frac{-b \pm \sqrt{D}}{2a} = \frac{-5 \pm \sqrt{2809}}{2 \cdot 3} = \frac{-5 \pm 53}{6}$$

$$x^2 \cdot \sqrt{x} = \frac{-5+53}{6} = 8 \text{ (kwadr.)} \vee x^2 \cdot \sqrt{x} = \frac{-5-53}{6} = -... \text{ (k.n.)}$$

$$x^4 \cdot x = x^5 = 64$$

$$x = \sqrt[5]{64} \text{ (voldoet).}$$

$$\boxed{\begin{array}{|c|c|} \hline 8^2 & 64 \\ \hline 5 \cdot \sqrt{64} \Rightarrow x & 2.29739671 \\ \hline x = \sqrt[5]{64} & 8 \\ \hline \end{array}}$$

G3h \square $(2x-1)^4 - 5(2x-1)^2 + 4 = 0$ (stel $(2x-1)^2 = t$)
 $t^2 - 5t + 4 = 0$
 $(t-4) \cdot (t-1) = 0$
 $(2x-1)^2 = 4 \vee (2x-1)^2 = 1$
 $2x-1 = \pm 2 \vee 2x-1 = \pm 1$
 $2x = 1 \pm 2 \vee 2x = 1 \pm 1$
 $x = 1 \frac{1}{2} \vee x = -\frac{1}{2} \vee x = 1 \vee x = 0.$

G4a \square $\frac{2x}{x-1} - 2 = 4$

$$\frac{2x}{x-1} = 6 = \frac{6}{1}$$

$$2x \cdot 1 = 6 \cdot (x-1)$$

$$2x = 6x - 6$$

$$-4x = -6$$

$$x = \frac{-6}{-4} = 1 \frac{1}{2} \text{ (voldoet).}$$

G4d \square $\frac{2x-1}{x+2} = \frac{x+4}{x-2}$

$$(2x-1) \cdot (x-2) = (x+2) \cdot (x+4)$$

$$2x^2 - 4x - x + 2 = x^2 + 4x + 2x + 8$$

$$x^2 - 11x - 6 = 0 \quad (a=1, b=-11 \text{ en } c=-6)$$

$$D = (-11)^2 - 4 \cdot 1 \cdot -6 = 145$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{11 \pm \sqrt{145}}{2 \cdot 1} = \frac{11 \pm \sqrt{145}}{2}$$

$$x = \frac{11+\sqrt{145}}{2} \text{ (voldoet)} \vee x = \frac{11-\sqrt{145}}{2} \text{ (voldoet).}$$

G4g \square $\sqrt{2-2x} = -2x \text{ (kwadrateren)}$

$$2-2x = (-2x)^2$$

$$0 = 4x^2 + 2x - 2$$

$$2x^2 + 1x - 1 = 0 \quad (a=2, b=1 \text{ en } c=-1)$$

$$D = 1^2 - 4 \cdot 2 \cdot -1 = 1 + 8 = 9$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{-1 \pm \sqrt{9}}{2 \cdot 2} = \frac{-1 \pm 3}{4}$$

$$x = \frac{-1+3}{4} = \frac{2}{4} = \frac{1}{2} \text{ (vold. niet)} \vee x = \frac{-1-3}{4} = \frac{-4}{4} = -1 \text{ (vold.).}$$

G5a \square $\begin{cases} 3x - 2y = -5 & |1 \\ -x + 5y = 32 & |3 \end{cases}$

$$\begin{cases} 3x - 2y = -5 \\ -3x + 15y = 96 \end{cases} +$$

$$13y = 91$$

$$y = 7$$

$$\begin{cases} -x + 5y = 32 \\ -x = -3 \end{cases} \Rightarrow -x + 35 = 32$$

$$x = 3.$$

G5b \square $\begin{cases} 4x + 2y = 14 & |3 \\ 5x - 3y = 45 & |2 \end{cases}$

$$\begin{cases} 12x + 6y = 42 \\ 10x - 6y = 90 \end{cases} +$$

$$22x = 132$$

$$x = 6$$

$$\begin{cases} 4x + 2y = 14 \\ 2y = -10 \end{cases}$$

G4h \square $3x = \sqrt{8x+1} \text{ (kwadrateren)}$

$$(3x)^2 = 8x + 1$$

$$9x^2 - 8x - 1 = 0 \quad (a=9, b=-8 \text{ en } c=-1)$$

$$D = (-8)^2 - 4 \cdot 9 \cdot -1 = 64 + 36 = 100$$

$$x = \frac{-b \pm \sqrt{D}}{2a} = \frac{8 \pm \sqrt{100}}{2 \cdot 9} = \frac{8 \pm 10}{18}$$

$$x = \frac{8+10}{18} = 1 \text{ (vold.)} \vee x = \frac{8-10}{18} = \frac{-2}{18} = -\frac{1}{9} \text{ (vold. niet).}$$

$$\boxed{\begin{array}{|c|c|} \hline 1 \Rightarrow x & 1 \\ \hline 3x & 3 \\ \hline \sqrt{8x+1} & 3 \\ \hline \end{array}}$$

$$\boxed{\begin{array}{|c|c|} \hline -1/9 \Rightarrow x & -1 \\ \hline 3x & -3 \\ \hline \sqrt{8x+1} & -3 \\ \hline \end{array}}$$

G6 \square $(-4, 42)$ op grafiek $\Rightarrow 42 = -32 + a \cdot 16 + b \cdot -4 + 6;$
 $(2, 12)$ op parabool $\Rightarrow 12 = 4 + a \cdot 4 + b \cdot 2 + 6.$

$$\begin{cases} 16a - 4b = 68 & |1 \\ 4a + 2b = 2 & |2 \end{cases}$$

$$\begin{cases} 16a - 4b = 68 \\ 8a + 4b = 4 \end{cases} +$$

$$24a = 72$$

$$a = 3$$

$$\begin{cases} 4a + 2b = 2 \\ 2b = -10 \end{cases} \Rightarrow 12 + 2b = 2$$

$$b = -5.$$

67a $\begin{cases} a + b = 150 \\ 8,6a + 7,0b = 1185 \end{cases} \quad | \begin{array}{l} 7 \\ 1 \end{array}$

$$\begin{array}{rcl} 7 \cdot 9 \cdot 150 & 1185 \\ 7 \cdot 150 & \\ -135 / -1,6 & 1050 \\ \hline 150 - Ans & 84,375 \end{array}$$

$$\begin{array}{l} a = 84,375 \\ \Rightarrow a + b = 150 \end{array} \Rightarrow 84,375 + b = 150$$

$$b = 65,625.$$

67b Stel hij neemt x ml van 15% en y ml van 30%.

$$\begin{cases} x + y = 600 \\ 15x + 30y = 13200 \end{cases} \quad | \begin{array}{l} 15 \\ 1 \end{array}$$

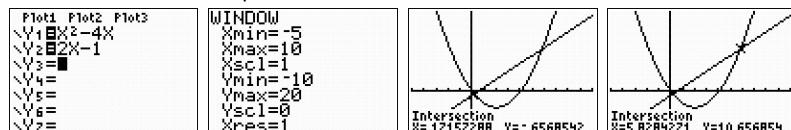
$$\begin{array}{rcl} 15x + 15y = 9000 & 13200 \\ 4200 / 15 & 280 \\ \hline 600 - 280 & 320 \end{array}$$

$$\begin{array}{l} y = 280 \\ \Rightarrow x + y = 600 \end{array} \Rightarrow x + 280 = 600$$

$$x = 320.$$

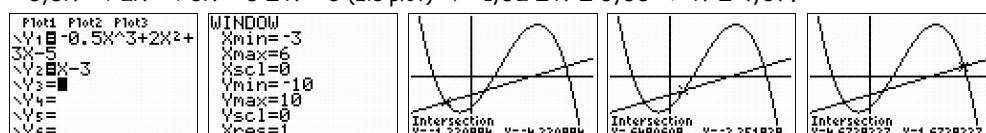
68a $x^2 - 4x = 2x - 1$ (intersect) $\Rightarrow x \approx 0,17 \vee x \approx 5,83.$

$x^2 - 4x > 2x - 1$ (zie plot) $\Rightarrow x < 0,17 \vee x > 5,83.$



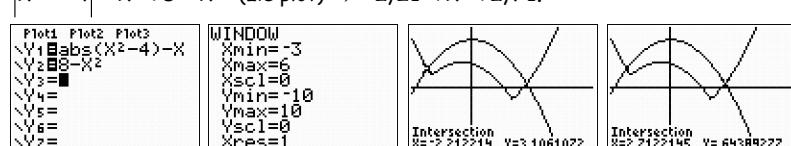
68b $-0,5x^3 + 2x^2 + 3x - 5 = x - 3$ (intersect) $\Rightarrow x \approx -1,32 \vee x \approx 0,65 \vee x \approx 4,67.$

$-0,5x^3 + 2x^2 + 3x - 5 \leq x - 3$ (zie plot) $\Rightarrow -1,32 \leq x \leq 0,65 \vee x \geq 4,67.$



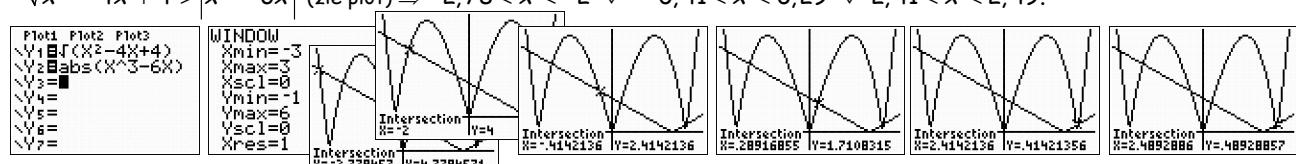
68c $|x^2 - 4| - x = 8 - x^2$ (intersect) $\Rightarrow x \approx -2,21 \vee x \approx 2,71.$

$|x^2 - 4| - x < 8 - x^2$ (zie plot) $\Rightarrow -2,21 < x < 2,71.$



68d $\sqrt{x^2 - 4x + 4} = |x^3 - 6x|$ (intersect) $\Rightarrow x \approx -2,78 \vee x = -2 \vee x \approx -0,41 \vee x \approx 0,29 \vee x \approx 2,41 \vee x \approx 2,49.$

$\sqrt{x^2 - 4x + 4} > |x^3 - 6x|$ (zie plot) $\Rightarrow -2,78 < x < -2 \vee -0,41 < x < 0,29 \vee 2,41 < x < 2,49.$



69a $px^3 + 2px^2 + x^2 + 2\frac{1}{4}x = px^3 + (2p+1)x^2 + 2\frac{1}{4}x = x \cdot (px^2 + (2p+1)x + 2\frac{1}{4}) = 0 \Rightarrow$

$x = 0 \vee px^2 + (2p+1)x + 2\frac{1}{4} = 0.$

Drie oplossingen als $px^2 + (2p+1)x + 2\frac{1}{4} = 0$ ($a = p \neq 0$, $b = 2p+1$ en $c = 2\frac{1}{4}$) twee oplossingen heeft.

$D = (2p+1)^2 - 4 \cdot p \cdot 2\frac{1}{4} = (2p+1)^2 - 9p.$

$D = 0 \Rightarrow (2p+1)^2 - 9p = 0$ (exact of intersect) $\Rightarrow p = \frac{1}{4} \vee p = 1.$

Drie oplossingen $\Rightarrow D > 0$ (zie plot) $\Rightarrow p < 0 \vee 0 < p < \frac{1}{4} \vee p > 1.$

($p = 0$ geeft $x^2 - 2\frac{1}{4}x = 0 \Rightarrow x \cdot (x - 2\frac{1}{4}) = 0 \Rightarrow x = 0 \vee x = 2\frac{1}{4} \Rightarrow 2$ oplossingen)

69b $2px^4 - px^3 + 5x^3 + 2x^2 = 2px^4 + (5-p)x^3 + 2x^2 = x^2 \cdot (2px^2 + (5-p)x + 2) = 0 \Rightarrow$

$$x = 0 \vee 2px^2 + (5-p)x + 2 = 0.$$

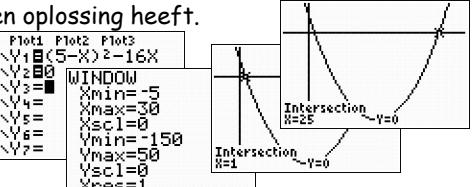
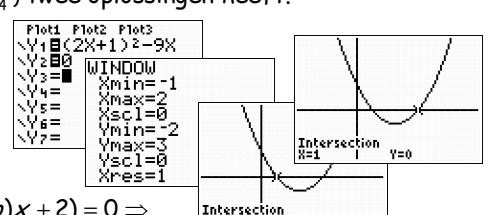
Eén oplossing als $2px^2 + (5-p)x + 2 = 0$ ($a = 2p \neq 0$, $b = 5-p$ en $c = 2$) geen oplossing heeft.

$D = (5-p)^2 - 4 \cdot 2p \cdot 2 = (5-p)^2 - 16p.$

$D = 0 \Rightarrow (5-p)^2 - 16p = 0$ (exact of intersect) $\Rightarrow p = 1 \vee p = 25.$

Eén oplossing $\Rightarrow D < 0$ (zie plot) $\Rightarrow 1 < p < 25.$

($p = 0$ geeft $5x^3 - x^2 = x^2 \cdot (5-x) = 0 \Rightarrow x = 0 \vee x = 5 \Rightarrow 2$ oplossingen)



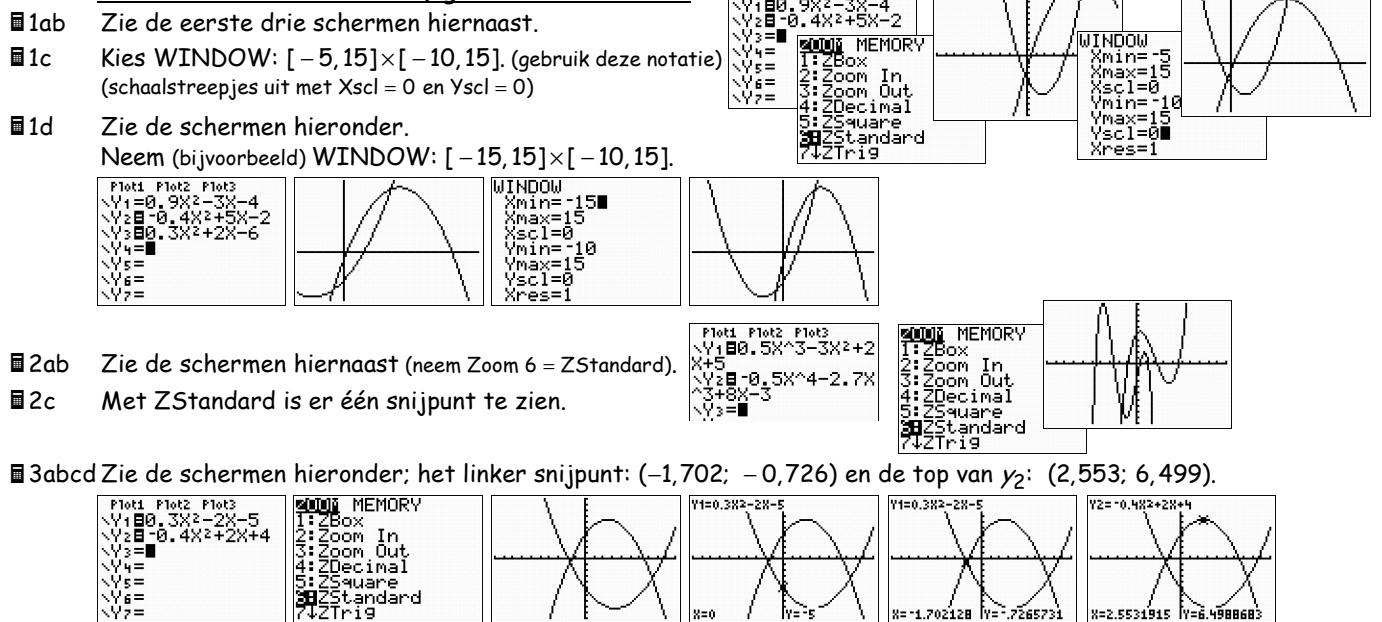
TI-84

1. Berekeningen op het basisscherm

■1a	$5,36^4 + 5 \times 1,47^2 \approx 836,19.$	$\boxed{5.36^{4+5*1.47^2} \\ 836.1944162}$	■1c	$1,8^2 : 3^5 \approx 0,01.$	$\boxed{1.8^2/3^5 \\ 0.01333333333}$
■1b	$\sqrt{34} + 6,5^3 \approx 280,46.$	$\boxed{\sqrt{34}+6.5^3 \\ 280.4559519}$	■1d	$11,5^2 + \sqrt{8,7} \approx 135,20.$	$\boxed{11.5^2+\sqrt{8.7} \\ 135.1995762}$
■2a	$\sqrt{12} + 3,51 \approx 6,97.$	$\boxed{\sqrt{12)+3.51 \\ 6.974101615}$	■2c	$\sqrt{21,8} : 3,51 \approx 1,33.$	$\boxed{\sqrt{21.8)/3.51 \\ 1.338212824}$
■2b	$\sqrt{12 + 3,51} \approx 3,94.$	$\boxed{\sqrt{12+3.51) \\ 3.938273734}$	■2d	$\sqrt{21,8 : 3,51} \approx 2,49.$	$\boxed{\sqrt{21.8 : 3.51) \\ 2.492152927}$
■3a	$-3,5^2 - 8 \times -3 = 11,75.$	$\boxed{-3.5^2-8*-3 \\ \boxed{11.75}}$	■3c	$-8,13^4 - \sqrt{5} : 1,6^3 \approx -4368,25.$	$\boxed{-8.13^4-\sqrt{5}:1.6^3 \\ -4368.254275}$
■3b	$\sqrt{8,91} - 3,1 \times 1,3^3 \approx -3,83.$	$\boxed{\sqrt{8.91}-3.1*1.3^3 \\ -3.825737689}$	■3d	$-8,1 \times 1,3^4 - 5,7^2 : -8 \approx -19,07.$	$\boxed{-8.1\times1.3^4-5.7^2:-8 \\ -19.07316}$
■4a	$(-5,7)^2 = 32,49.$	$\boxed{(-5.7)^2 \\ 32.49}$	■4c	$-5,7^2 = -32,49.$	$\boxed{-5.7^2 \\ -32.49}$
■4b	$(-1,8)^4 = 10,4976.$	$\boxed{(-1.8)^4 \\ 10.4976}$	■4d	$-1,8^4 = -10,4976.$	$\boxed{-1.8^4 \\ -10.4976}$
■5a	$\frac{118-53}{53} \times 100 \approx 122,6.$	$\boxed{(118-53)/53*100 \\ 122.6415094}$	■5c	$\frac{1371-862}{128} \approx 4,0.$	$\boxed{(1371-862)/128 \\ 3.9765625}$
■5b	$\frac{100}{352 \times 1,23} \approx 0,2.$	$\boxed{100/(352*1.23) \\ .2309682188}$	■5d	$\frac{1283-1827}{1827} \times 100 \approx -29,8.$	$\boxed{(1283-1827)/1827 \\ *100 \\ -29.7755884}$
■6a	$\frac{118,6}{8,3^2-5,6} \approx 1,87.$	$\boxed{118.6/(8.3^2-5.6) \\ 1.87391373}$	■6c	$\frac{-1,31+8,3 \times 7,05}{21,3^2-7,5^3} \approx 1,80.$	$\boxed{(-1.31+8.3\times7.05) \\ /(21.3^2-7.5^3) \\ 1.798051234}$
■6b	$\frac{5,93+\sqrt{23}}{8,41-3\sqrt{15}} \approx -3,34.$	$\boxed{(5.93+\sqrt{23})/(8. \\ 41-3*\sqrt{15}) \\ -3.342473829}$	■6d	$\frac{3,88^2+4,26^3}{1+\sqrt{5,6}-2,9^2} + 7,4^3 \approx 386,91.$	$\boxed{3.88^2+4.26^3 \\ 1+\sqrt{5.6}-2.9^2+7 \\ 386.9109378}$
■7a	$\frac{2}{3} + \frac{1}{4} = \frac{11}{12}.$	$\boxed{\frac{2}{3}+1/4 \\ 11/12}$	■7c	$20 \times 1\frac{3}{7} = \frac{200}{7}.$	$\boxed{20*(1+3/7)\rightarrow\text{Frac} \\ 200/7}$
■7b	$(1\frac{2}{9})^2 = \frac{121}{81}.$	$\boxed{(1\frac{2}{9})^2 \\ 121/81}$	■7d	$19 \times 2\frac{1}{3} - 8 \times 2\frac{4}{7} = \frac{499}{21}.$	$\boxed{19*(2+1/3)-8*(2+ \\ 4/7)\rightarrow\text{Frac} \\ 499/21}$
■8a	$8\frac{3}{5} : 2\frac{1}{4} = \frac{172}{45}.$	$\boxed{(8+3/5)/(2+1/4)\rightarrow \\ \text{Frac} \\ 172/45}$	■8c	$(3\frac{1}{6} - 2\frac{1}{7})^2 = \frac{1849}{1764}.$	$\boxed{((3+1/6)-(2+1/7))^2\rightarrow\text{Frac} \\ 1849/1764}$
■8b	$(3\frac{1}{6} - 2\frac{1}{5}) : 2\frac{1}{5} = \frac{29}{66}.$	$\boxed{(3\frac{1}{6}-2\frac{1}{5})\rightarrow\text{Frac} \\ 29/66}$	■8d	$21 : 2\frac{3}{7} = \frac{147}{17}.$	$\boxed{21:(2\frac{3}{7})\rightarrow\text{Frac} \\ 147/17}$
■9a	$(1\frac{2}{3})^2 = \frac{25}{9} \cdot \boxed{(1+2/3)^2\rightarrow\text{Frac} \\ 25/9}$	■9b	$(-2\frac{3}{7})^4 = \frac{83521}{2401}.$	■9c	$5 : 1\frac{1}{3} = \frac{15}{4} \cdot \boxed{5*(1+1/3)\rightarrow\text{Frac} \\ 15/4}$

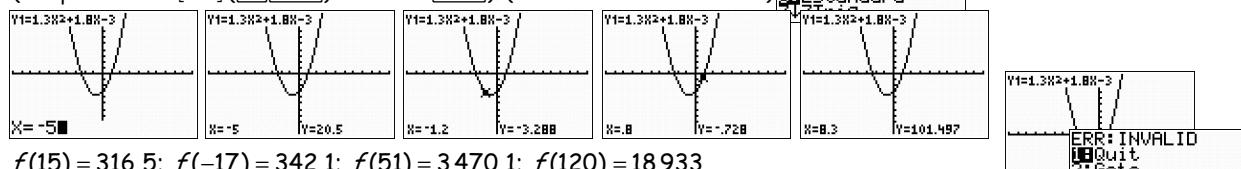
TI-84

2. Formules, grafieken en tabellen

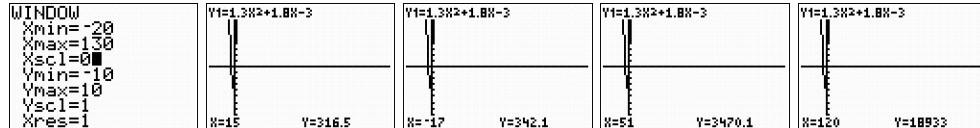


■ 4a Zie de schermen hiernaast.

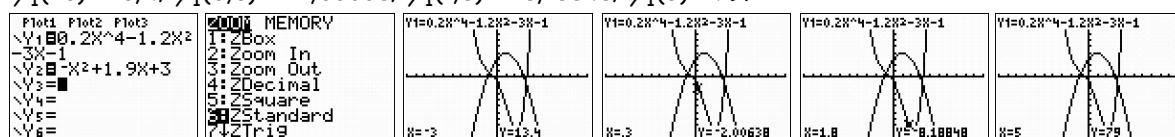
■ 4b $f(-5) = 20,5$; $f(-1,2) = -3,288$; $f(0,8) = -0,728$; $f(8,3) = 101,497$.
(de optie value in [CALC] ([2nd][TRACE]) werkt als [TRACE]). (zie de schermen hieronder)



■ 4cde $f(15) = 316,5$; $f(-17) = 342,1$; $f(51) = 3470,1$; $f(120) = 18933$.



■ 5a $y_1(-3) = 13,4$; $y_1(0,3) = -2,00638$; $y_1(1,8) = -8,18848$; $y_1(5) = 79$.



■ 5b $y_2(-3) = -11,7$; $y_2(0,3) = 3,48$; $y_2(1,8) = 3,18$; $y_2(5) = -12,5$.

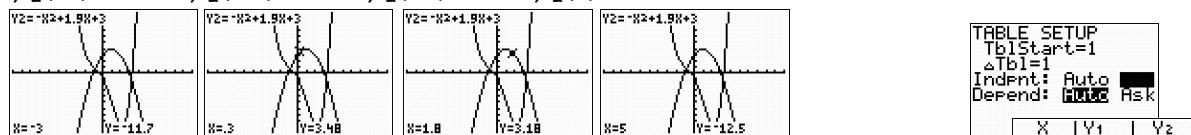
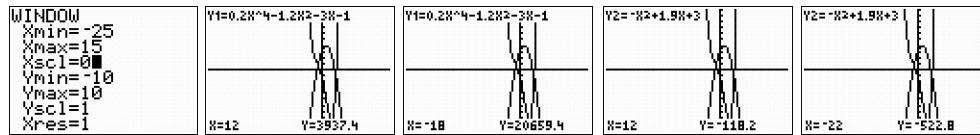


TABLE SETUP
 $\text{TblStart}=1$
 $\Delta\text{Tbl}=1$
 IndPnt: Auto
 Depend: Hulte Rsk

X	y_1	y_2
-3	13,4	-11,7
0,3	-2,00638	3,48
1,8	-8,18848	3,18
5	79	-12,5
12	20659,4	-522,8
18	3937,4	-118,2
22	46335	-522,8

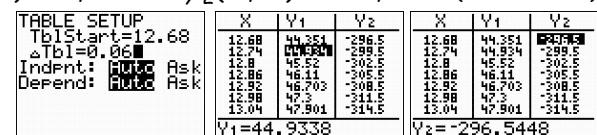
■ 5cd $y_1(12) = 3937,4$; $y_1(-18) = 20659,4$; $y_2(12) = -118,2$; $y_2(-22) = -522,8$.



■ 6ab Zie hieronder: $y_1(4,15) = -1,83875$.

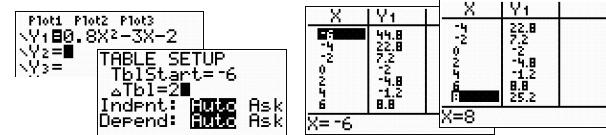


■ 6c $y_1(12,74) = 44,9338$ en $y_2(12,68) = -296,5448$. (zie hieronder)

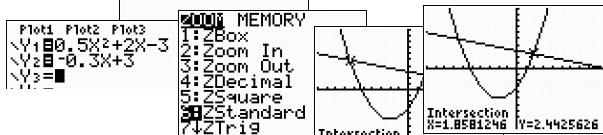


■ 7 Haal de antwoorden uit de tabel hiernaast.

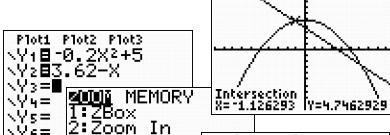
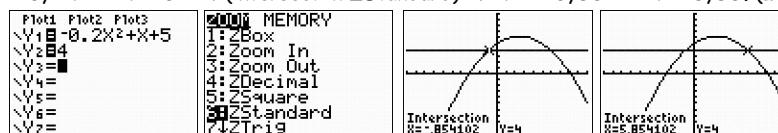
x	-6	-4	-2	0	2	4	6	8
$f(x)$	44,8	22,8	7,2	-2	-4,8	-1,2	8,8	25,2



■ 8 $y_1 = 0,5x^2 + 2x - 3$ en $y_2 = -0,3x^2 + 3$. (zie hiernaast)
optie intersect $\Rightarrow S_1(-6,46; 4,94)$ en $S_2(1,86; 2,44)$.

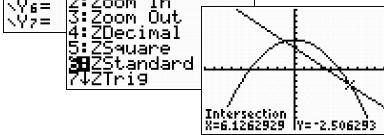
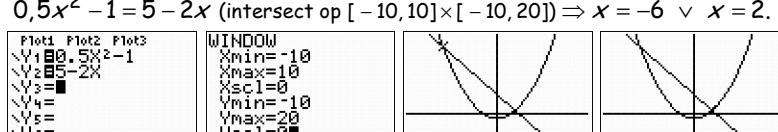


■ 9 $-0,2x^2 + x + 5 = 4$ (intersect in ZStandard) $\Rightarrow x \approx -0,85$ v $x \approx 5,85$. (zie de schermen hieronder)



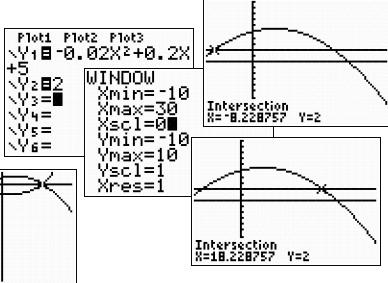
■ 10a $-0,2x^2 + 5 = 3,62 - x$ (intersect in ZStandard) $\Rightarrow x \approx -1,13$ v $x \approx 6,13$.

■ 10b $0,5x^2 - 1 = 5 - 2x$ (intersect op $[-10,10] \times [-10,20]$) $\Rightarrow x = -6$ v $x = 2$.

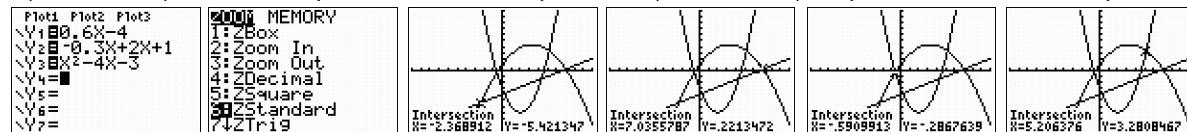


■ 10c $-0,02x^2 + 0,2x + 5 = 2$ (intersect op $[-10, 30] \times [-10, 10]$) $\Rightarrow x \approx -8,23 \vee x \approx 18,23$.

■ 10d $-0,4x^3 - 10 = 5 + 4x - 2x^2$ (intersect op $[-8, 5] \times [-100, 10]$)
 $x \approx -5,59 \vee x \approx -2,31 \vee x \approx 2,90$. (zie de schermen hieronder)



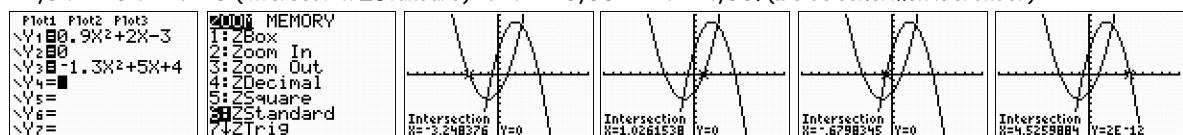
■ 11ab $0,6x - 4 = -0,3x^2 + 2x + 1$ (intersect in ZStandard) $\Rightarrow x \approx -2,37 \vee x \approx 7,04$. (zie de schermen hieronder)



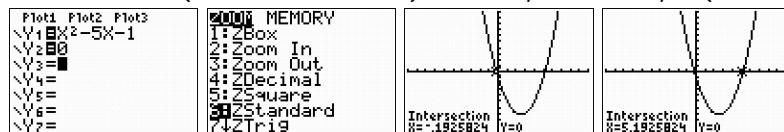
■ 11c $0,6x - 4 = -0,3x^2 + 2x + 1$ (intersect in ZStandard) $\Rightarrow x \approx -0,59 \vee x \approx 5,21$. (zie de schermen hierboven)
(het is niet nodig om grafieken uit te zetten; kies met of bij First curve? en/of Second curve? de juiste formules)

■ 12abc $0,9x^2 + 2x - 3 = 0$ (intersect in ZStandard) $\Rightarrow x \approx -3,25 \vee x \approx 1,03$. (zie de schermen hieronder)

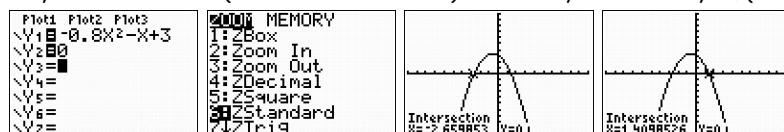
■ 12d $-1,3x^2 + 5x + 4 = 0$ (intersect in ZStandard) $\Rightarrow x \approx -0,68 \vee x \approx 4,53$. (zie de schermen hieronder)



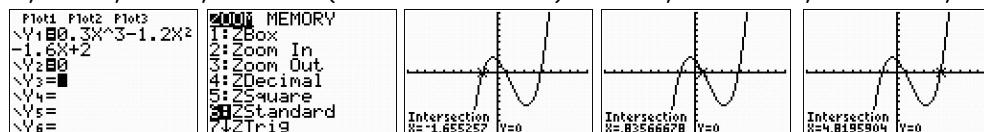
■ 13a $x^2 - 5x - 1 = 0$ (intersect in ZStandard) $\Rightarrow x \approx -0,19 \vee x \approx 5,19$. (zie de schermen hieronder)



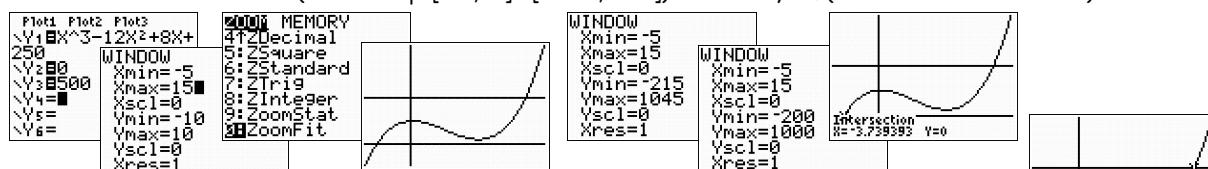
■ 13b $-0,8x^2 - x + 3 = 0$ (intersect in ZStandard) $\Rightarrow x \approx -2,66 \vee x \approx 1,41$. (zie de schermen hieronder)



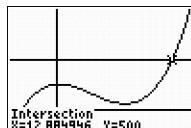
■ 13c $0,3x^3 - 1,2x^2 - 1,6x + 2 = 0$ (intersect in ZStandard) $\Rightarrow x \approx -1,66 \vee x \approx 0,84 \vee x \approx 4,82$. (zie de schermen hieronder)



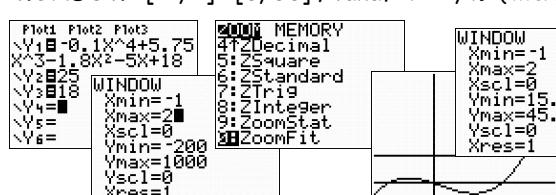
■ 14ab $x^3 - 12x^2 + 8x + 250 = 0$ (intersect op $[-5, 15] \times [-200, 1000]$) $\Rightarrow x \approx -3,74$. (zie de schermen hieronder)



■ 14c $x^3 - 12x^2 + 8x + 250 = 500$ (intersect) $\Rightarrow x \approx 12,88$. (zie het scherm hiernaast)



■ 15a WINDOW: $[-1, 2] \times [0, 50]$; vanaf $x \approx 1,49$ (intersect en plot).



■ 15b $x \approx -0,78 \vee x \approx 1,11$ (intersect).

